DISCUSSION

Disc. 111-S60/From the July-August 2014 *ACI Structural Journal*, p. 731

Depth Effect in Deep Beams. Paper by David B. Birrcher, Robin G. Tuchscherer, Matt Huizinga, and Oguzhan Bayrak.

Discussion by Rafael Alves de Souza and Sergio Breña

ACI member, Full Professor, Department of Civil Engineering, State University of Maringá, Maringá, Brazil; ACI member, Associate Professor, Department of Civil and Environmental Engineering, University of Massachusetts Amherst, Amherst, MA.

The authors have presented interesting results concerning the depth effect of deep beams. Strength results indicate that the shear capacity of deep beams is governed by the strength of the nodal regions that are not directly proportional to the member depth. Despite quality research, additional issues are discussed for clarity.

BACKGROUND

In this section, the authors have presented the experimental results of three deep beam depth effect studies by Walraven and Lehwalter,[8] Tan and Lu,[9] and Zhang and Tan.[11] Walraven and Lehwalter[8] and Zhang and Tan[11] scaled the length of the bearing plates to the depth of the members with different results. While Walraven and Lehwalter[8] observed a depth effect, Zhang and Tan[11] did not observe a discernible reduction in the normalized shear strength of the tested deep beams. The source of the observed differences is likely due to the high bearing stress at the single applied load at midspan of the Walraven and Lehwalter[8] tests. For example, Walraven and Lehwalter[8] have used a three-point bending instead of the four-point bending test selected by Zhang and Tan.[11] Therefore, the bearing stress at the load plate was twice as high as those at the support plates for the Walraven and Lehwalter[8] tests. The authors concluded that the length of the bearing plates on the behavior of deep beams could be the main parameter for explaining depth effects. They conducted some experimental research guided by this hypothesis that confirmed the process.

That hypothesis was a strong factor in explaining the depth effect results. The depth effect in deep beams, however, is not as simple and cannot be attributed to the isolated effect. As mentioned by Zhang and Tan,[11] whether size effect occurs in deep beams remains questionable among researchers. Experimental data are relatively scarce on deep beams with geometrically varied beam size and reinforcement, while maintaining the same shear span-depth ratio (a/d) and comparable concrete strength.

Based on the research, it appears that the observed decrease in shear strength with the increase in deep beam depth could only be attributed to the length of the bearing plates. Magnitude of loading and the dimensions of support plates have a significant effect on shear strength of deep beams. Other factors have been noted as having possible effects on deep beams, for example: 1) amount of longitudinal reinforcement (level of strains in longitudinal reinforcement considerably affects the strength of the diagonal struts); 2) amount of transverse reinforcement (contributes to crack control of bottle-shaped struts and plays a diminution of the level of strains of longitudinal reinforcement); 3) aggregate size and aggregate interlock (for beams without transverse reinforcement, aggregate interlock has a great contribution, and the relation between the aggregate size and depth appears to have some influence in the response); 4) out-of-plane effects (relation between depth and width, especially for very slim deep beams); and 5) shear span-depth ratio (beams with low-relation a/d seem to offer high shear stress for deep beams).

Tan et al.[19] and Collins and Kuchma[5] have shown that the tensile force at the bottom longitudinal steel was reduced owing to the presence of web reinforcement. Bažant and Sun[20] have shown that the presence of web reinforcement can mitigate the variation of shear strength in deep beams. Also, as mentioned by Tan et al.,[21] the modified compression field confirms that size effect depends on the spacing of web reinforcement rather than solely on the overall beam depth.

Tan et al.[21] concluded that when only the loading plate width is increased proportionally with beam size, there is still some size effect on shear strength, although weaker. They also concluded that in deep beams, after diagonal crack formation, the arch action dominates and a significant portion of the load is transferred directly from the loading point to the support by diagonal compression struts. The size effect, therefore, depends on the extent to which the arch action (strut effectiveness) is mobilized. This mobilization is dependent on the loading and support plates (strut geometry) as well on the spacing and diameter of web reinforcement transverse to the inclined struts (strut boundary conditions).

Bažant and Kazemi[7] argued that size effect induced by energy release would be evidenced by greater crack propagation rates for larger-sized beams. In the discussers' opinion, this effect is mainly related to the aggregate size and the transverse reinforcement provided with increasing depth.

In the discussers' opinion, to mitigate the depth effect, it might also be interesting to control the aggregate size (keep a constant proportion between aggregate size and depth) and control the horizontal and vertical reinforcement proportion (calculate the grid reinforcement based on the transversal tensile strains induced in the bottle shape strut; for example, higher beams have more probability to develop bottle-shaped struts despite the fact that length bearing is kept constant). One should also consider the load plate and bearing dimensions.

Kotsovos and Pavlovic[22] argued that it might be difficult to prevent the occurrence of out-of-plane actions when the beam cross section is slim (inducing buckling failure) and that small stresses induced by unintended out-of-plane actions could have a significant effect on beam strength.

The question is, based on these additional effects that could have an influence on the depth effect in deep beams, could the authors provide some additional comments?

EXPERIMENTAL INVESTIGATION AND DISCUSSION RESULTS

For the specimens with a/d of 1.2 and 1.85, at approximately 20 to 25% of the maximum applied load, the first diagonal shear crack formed in the tested regions.

Comparing these values with other available results in the literature,[11,19] it is possible to verify that the obtained values are lower. Zhang and Tan[11] have tested deep beams with an a/d of 1.1 while Tan et al[19] tested deep beams with an a/d of 0.75. Do the authors believe that span-depth ratio can affect the diagonal cracking load, even though transverse reinforcement is inactive at that load level?

An analysis of the tested specimens was estimated using strut-and-tie modeling recommendations developed by Tuchscherer et al.[13] At that time, this procedure was considered more simple, accurate, and conservative as the strut-and-tie provisions, as well as the specifications of ACI 318-08 Appendix A,[14] AASHTO LFRD 2008,[15] and *fib*.[16] Recall that Zhang and Tan[11] recommended strut-and-tie models as a solution, based on the influence of loading/bearing plates; therefore, what is the difference between the model proposed by Tuchscherer et al.[13] and that of Zhang and Tan?[11] The experimental program conducted by the authors appeared influenced by the research previously conducted by Zhang and Tan.[11]

As previously discussed, the premature splitting caused by transverse tension in the bottle-shaped strut was indirectly accounted for by including minimum web reinforcement of 0.2% in each orthogonal direction in all test specimens—an amount that should ensure the full design strength of a diagonal strut. It was concluded that more experimental evidence is required to confirm this assumption. Higher depths should require much more than 0.2% web reinforcement to better control the transverse tensile stresses developed in bottle-shaped struts. Although there is some evidence that web reinforcement does not affect diagonal cracking, it does slow down propagation of diagonal cracks toward the top and bottom nodal zones, enhancing the strut capacity.[21]

Note that no mention was made of displacement obtained in the tests. Could the authors provide plots including nominal shear stress versus midspan displacement or normalized shear stress versus midspan displacement? They would be useful in demonstrating how depth effect could be mitigated when the loading and bearing plate are judiciously selected.

CONCLUSIONS

In the discussers' opinion, the depth effect in deep beams depends on the loading and bearing plate dimensions and the strut boundary. That would be the amount of mesh reinforcement provided for controlling bottle-shaped struts and the level of deformation of the main tie (longitudinal bottom reinforcement). Additional effects for out-of-plane, aggregate size, a/d relation, and grid reinforcement should be further explored to improve the understanding of deep beams shear behavior. Hopefully, this simplifies the worldwide issue regarding the case of slender beams.[23]

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AUTHORS' CLOSURE

The authors appreciate the interest of the discussers in this paper.

The primary factors that contribute to deep-beam strength according to the authors are identified in the paper. Those factors are captured by the strut-and-tie model design checks presented in the paper. They include compressive stresses in the nodal regions, strength of the primary tension tie, and treatment of transverse tension in the bottle-shaped strut.

A relationship between shear span-depth ratio and diagonal cracking loads has been observed as presented in Birrcher et al.[1]

The discussers are encouraged to compare the strut-and-tie models of Tuchscherer et al.[3] and Zhang and Tan[11] to assess their similarities and differences.

Presenting the plots of shear stress versus displacement was not required to meet the goals of the paper.

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Shear Strength of Reinforced Concrete Beams. Paper by Wu Wei Kuo, Thomas T. C. Hsu, and Shyh Jiann Hwang.

Discussion by Jian Yuan

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Based on the shear design method of prestressed concrete (PC) beams (UH method) and on the concept of effective web area by Tureyen and Frosch,[11] a shear design method for reinforced concrete (RC) beams that was consistent with that for PC beams was presented in this paper. Some of the findings are interesting, but some research is still questionable and worthy of further discussion.

Instead of using the web area $b_w d$ as a variable in the V_c term of the UH method, Eq. (13) used the web compression area $b_w c$, but all other parameters remained the same. Meanwhile, compared with Eq. (11), the coefficients in Eq. (16) were modified. Even though these proposed equations were verified using 313 RC beams available in the literature, the derivation process for such changes still need to be supplemented.

Involving the rationality of Eq. (15) and (16), this paper did not conduct thorough analysis. Moreover, the limiting conditions for Eq. (15) were not given. Assuming shear failure occurs in a beam with stirrups, if $(d/s - 1)$ falls below zero, will the stirrups play a negative role?

To compare the results of V_s calculated by the proposed method and ACI 318-11, test data of 74 pairs of beams has been obtained here by considering 13 papers available in the literature.[10,21,23,25,29,30,33-37,40,45] These test specimens contain beams with stirrups and that of the same beam without stirrups, which is called reference beam. Every reference beam

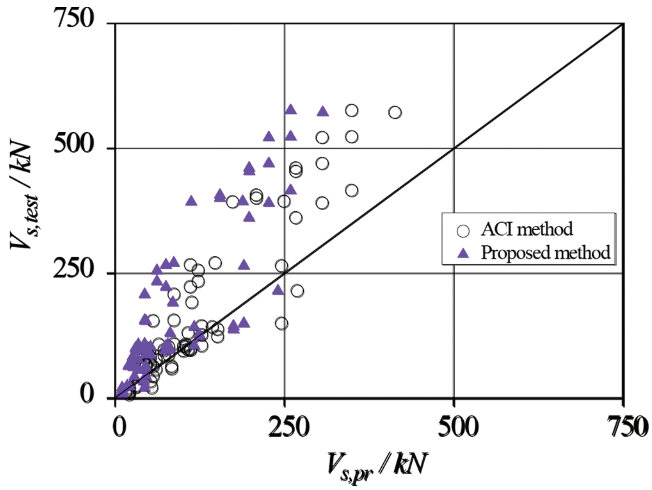

Fig. 7—Validation of predicting model (V_s) with test data.

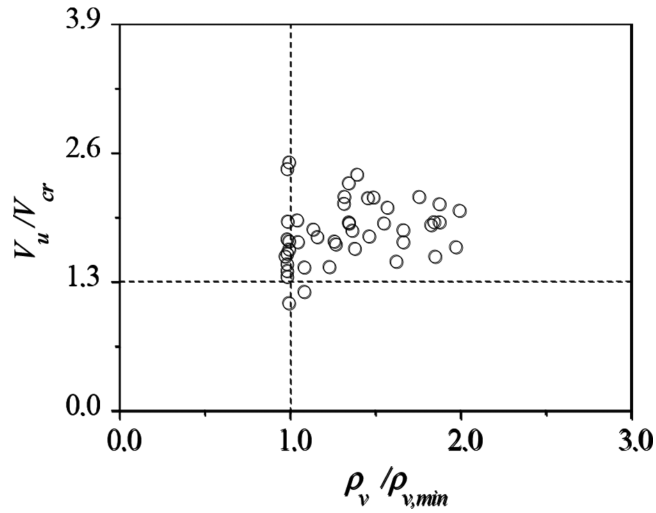

Fig. 8—Validation of amount of web reinforcement with test data.

is followed by its similar beams with stirrups, which can be more than one. The stirrup spacing in all these specimens is less than $d/2$, which can meet the requirements prescribed by ACI 318-11.When the measured shear strength of a beam with stirrups, $V_{u,test}$, and that of the reference beam without stirrups, $V_{c,test}$, are given, it is possible to obtain an experimental value of shear strength increase due to stirrup inclusion: $V_{s,test} = V_{u,test} - V_{c,test}$. The obtained predicted capacity $V_{s,pr}$ versus the test capacity $V_{s,test}$ is shown in Fig. 7. In this figure, a point representing ($V_{s,pr}$,$V_{s,test}$) that is above the 45-degree line indicates that the predicting model is conservative, while a point below the line indicates that the predicting model is unconservative. Even though the proposed method is a little more conservative than ACI method, as shown in Fig. 7, the ACI method is also reasonable.

Based on Table 4 and Fig. 5, the authors argued that both the ACI method and the proposed method overestimated the shear strengths of a number of specimens in Kain's Valley within the range of a/d from 2.5 to 4.0, and the minimum shear reinforcement for specimens in this region should be twice the amount required by ACI 318-11. In reality, V_{test}/V_{ACI} and V_{test}/V_{prop} cannot reflect whether the minimum shear reinforcement in ACI 318-11 is reasonable, because V_{ACI} and V_{prop} are the predicted shear strength rather than experimental results. The discusser sorts test data of 47 beams with stirrups from the literature[8,10,21,23,25,29,32,33,36,37,40] with a range of a/d from 2.5 to 4.0 and a $\rho_v/\rho_{v,min}$ ratio between 0.97 and 2.0, including shear forces corresponding to diagonal tension cracking (V_{cr}) and failure (V_u). Figure 8 represents the amount of stirrups provided in tested beams over the minimum amount specified by ACI 318-11 versus the reserve shear strength index (V_u/V_{cr}). Ozcebe et al.[32] suggested that to ensure an adequate margin of safety, the value of the index should be greater than 1.30. This condition is satisfied (Fig. 8) by the minimum amount of web reinforcement required by ACI 318-11.

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AUTHORS' CLOSURE

The authors wish to thank the discusser for providing additional research information and his constructive comments. The discusser made four points in his discussion. First, he pointed out that the equations for $V_{n,max}$ are different for prestressed concrete (PC) beams and reinforced concrete (RC) beams. This difference required explanations. Second, the discusser raised the question of what to do in Eq. (15) when ($d/s - 1$) falls below zero. Third, the discusser presented Fig. 7. In this figure, the predicted values of steel contribution ($V_{s,pr}$) by the proposed method as well as those by ACI 318-11 were compared with the experimental results ($V_{s,test}$). Fourth, the adequacy of the ACI 318-11 minimum amount of shear reinforcement was examined in Fig. 8 by plotting V_u/V_{cr} versus $\rho_v/\rho_{v,min}$. The authors address these four points of discussion in the following paragraphs.

1. For PC beams, $V_{n,max}$ of the UH method is taken as $1.33\sqrt{f'_c}b_wd$ (Eq. (11)). Whereas, the authors proposed that $V_{n,max}$ for RC beams be lowered to $1.1\sqrt{f'_c}b_wd$ (Eq. (16)). In the authors' opinion, the axial compression acting on the PC beams should make $V_{n,max}$ of PC beams larger than that of RC beams. The lowered coefficient in Eq. (16) was calibrated using the RC test data collected. Taking this opportunity, the authors would like to correct a typographical error in Eq. (16). In $V_{n\ max}$ $13\sqrt{f'_c}b_wd$ (f'_c in MPa), the unit of MPa should be replaced by psi.*

2. The discusser asked that, if ($d/s - 1$) falls below zero, will the stirrups play a negative role? The answer is "no." At worst, the stirrups should have no adverse effect. For beams with stirrups conforming to the detailing rule of $s < d/2$, the value of ($d/s - 1$) is always positive. For clarity, however, the authors would like to define $(d/s - 1) \geq 0$ in order to eliminate the negative value of ($d/s - 1$).

3. The discusser provided a good comparative work in Fig. 7. It can be seen that quite a few points predicted by ACI 318-11 fall below the 45-degree line, which indicates the ACI method is unconservative. This is the reason why the authors propose Eq. (15) based on the concept of minimum shear strength.

4. Fig. 8 shows clearly that when $\rho_v/\rho_{v,min}$ approaches unity, the shear strength V_u is close to the cracking value of V_{cr}. To avoid this circumstance, the authors suggested the increase of minimum shear reinforcement required by ACI 318-11 in the region of Kani's Valley.

*Editor's note: The PDF of the original paper was been corrected and is accessible via **www.concrete.org**.

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Experimental Evaluation of Strut-and-Tie Model of Indeterminate Deep Beam. Paper by D. B. Garber, J. M. Gallardo, G. D. Huaco, V. A. Samaras, and J. E. Breen.

Discussion by Rafael Alves de Souza and Sergio Breña

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The authors have presented the experimental results of statically indeterminate deep beams using the strut-and-tie modeling (STM) procedures available in ACI 318-08. Despite the important effort providing new experimental results, some additional issues should be discussed to fully understand and complement the procedures used in this very interesting paper. The authors are to be complimented by their effort producing experimental results for statically indeterminate structures designed using strut-and-tie models.

INTRODUCTION

In this section, the authors have mentioned that, while size does have an effect on the shear behavior of unreinforced specimens, it has been suggested that in reinforced concrete specimens, a more gentle size effect may be expected. This affirmation was based on the study conducted by Ley et al. (2007), who conducted experiments using small-scale specimens designed using STM at 1:10.5 and 1:6 scales.

In the discussers' opinion, the problem concerning scale effects in deep beams in the study was treated as a minor problem and this issue should deserve more in-depth reflection. Available experimental data (Walraven and Lehwalter 1994; Tan and Lu 1999) have shown that when depth is increased, a decrease in the shear strength is expected. While a judicious selection of the load and bearing plates (Bircherr et al. 2014; Zhang and Tan 2007; Tan et al. 2008) may mitigate the depth effect, there is still extensive discussion whether or not this effect is present in deep beams. In the discussers' opinion, small-scale specimens will present a more gentle size effect based on the fact that unintended out-of-plane effects will be less critical. However, the aggregate size will probably have a strong influence in the shear strength response of small-scale specimens. Could the authors, based on their experimental observations, provide some additional comments on these issues?

RESEARCH SIGNIFICANCE

In this section, the authors have mentioned that there has been little experimental validation of STM for statically indeterminate structures. While this information is true, no references were furnished to the readers. In this way, the discussers would like to contribute references highlighting some important researchers conducted in the field: Rogowsky and MacGregor (1983, 1986); Rogowsky et al. (1986); Ashour (1997); Maxwell and Breen (2000); Ashour and Rishi (2000); and Kuchma et al. (2008).

EXPERIMENTAL PROCEDURE

In the subsection "Design process," the authors mentioned that, with the purpose of designing an appropriate model for the concrete members, four strut-and-tie models (A, B, C, and D) were developed by four independent groups. Then the cited models were sketched based on linear finite element analysis (FEM)—that is, based on the elastic flow of stresses—giving rise to the truss models presented in Fig. 3. However, in subsection "Reinforcement layout," it is not very clear how the design forces in the strut and ties were obtained using a simple elastic truss analysis of the proposed strut-and-tie models.

Assuming that the degree of indeterminacy of a plane truss is given by $D = n + r - 2j$, where D is the degree of indeterminacy, n is the number of members, r is number of external restraints, and j is the number of joints, one may show that both Models B and C presented in Fig. 3 have at least one degree of indeterminacy if the boundary conditions presented in Fig. 2 are considered. This fact would lead to the necessity of defining the stiffness of the struts and ties to obtain the real forces in the members, unless one assumes that the horizontal reaction in the pinned support is zero due to the lack of horizontal external actions.

As observed by Tjhin and Kuchma (2002), there is little guidance available for evaluating the relative stiffness of members in a statically indeterminate strut-and-tie model and, as a result, the designer is unsure how to determine the distribution of forces in these types of trusses. The classical way to handle a statically indeterminate case is to employ the so-called plastic truss method, where one may assume that the heaviest ties have yielded and the truss becomes statically determinate. Another way is to decompose the statically indeterminate truss into several statically determinate trusses.

Could the authors better explain how they overcame the difficulties for determining the design forces in their trusses based on the internal indeterminacy for the proposed trusses? Also, how they overcame the fact that, near the square openings, there were some unstable regions (rectangular regions instead of triangular regions) demanding at least stabilizers (elements with zero forces but necessary to form stable trusses by forming adequate triangular regions)?

The authors mentioned that two layers of 14-gauge welded wire reinforcement were provided in the specimens in accordance with ACI 318-08 Appendix A.3.3.1 to allow for the stress spreading of bottle-shaped struts and widening of any shrinkage cracks. No mention was made, however, regarding the final area of this mesh reinforcement. Do the authors believe that the minimum mesh reinforcement of 0.3%, as prescribed by Appendix A.3.3.1, is adequate for their small-scale specimens? In the discussers' opinion, this minimum mesh area may be in excess for the small specimens tested and probably could capture a great parcel of the total loading. Could the authors please give more information regarding the mesh reinforcement adopted as well as their opinion concerning this possible contribution of the mesh reinforcement in the total strength of the specimens?

In the discussers' opinion, a separated small specimen containing only the mesh reinforcement should be tested to check the effectiveness of the proposed strut-and-tie models. In the future, authors are encouraged to quantify the real strength contribution of the selected mesh reinforcement, at least by means of nonlinear finite element analysis.

In the subsection "Testing apparatus," it can be observed that steel plates and neoprene bearing pads have been used for each one of the supports (three roller supports). However, the apparatus presented in Fig. 5 leads to different boundary conditions from that one presented in Fig. 2 (two rollers and one pinned support). Despite the fact that no horizontal forced is acting in the deep beams, why did the authors not use the pinned support and selected neoprene bearings?

Unfortunately, no mention was made regarding the strains of the main ties. Do the authors have some results of the strains? If so, could the authors provide some information regarding the strains?

EXPERIMENTAL RESULTS AND DISCUSSION

In the subsection "Performance of specimens," Table 2 shows that all of the measured failure loads were significantly higher than the factored design load (214 kN), reflecting the conservative, lower-bound nature of STM. Once STM is a lower-bound (static or equilibrium) method of limit analysis, and reinforcement steel must be designed for yielding before concrete crushes, a failure load of at least 285.33 kN could be expected for the tested specimens, if one considers the safety factors usually used in ACI 318. The mentioned minimum failure load may be obtained by dividing the factored design load by an assumed strength reduction factor ϕ of 0.75 for the reinforcement.

Taking this fact into consideration and observing that additional mesh reinforcement was also provided to the specimens, Specimen A did not seem to present an explicit redistribution due to the statically indeterminate system. As mentioned by the authors, in a statically indeterminate system, a local failure in one path may cause load redistribution but not a global failure, and two local failures are required for a global failure mechanism to form. Specimen A presented a reason measured/design load of 1.43 in Table 2, while a minimum reason measured/design load raised by the discussers would be at least 1.33. Could the authors estimate why Specimen A had the lower strength among the specimens?

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AUTHORS' CLOSURE

The authors would like to express their deepest gratitude to the discussers for their thoughtful comments and analysis of the paper. The discussers introduce several different concerns and comments presented by section, which will be addressed in kind.

INTRODUCTION

The discussers bring up the problem concerning size effect in reinforced concrete members, specifically in deep beams. The problem of size effect in deep beams is not a minor problem, but it was not the focus of this testing program. The authors' intention of this discussion was to remind readers of the applicability of strut-and-tie modeling for use on any size specimens (that is, results from this study on small-scale specimens could be reasonably applied to similar, larger-scale specimens). As the discusser mentions, when proportional size increases are made to specimen and bearing geometries, there will be "no adverse effect with increasing depth" (Birrcher et al. 2014). As only one size of specimen was tested in this study, the authors have no additional experimental input with regard to the issue of size effect or the discussers' stated opinions.

RESEARCH SIGNIFICANCE

The authors thank the discussers for highlighting the aforementioned research conducted in the area of continuous deep beams and propped cantilevers; these studies are valuable references on the subject.

It is important to mention that, despite the existing literature mentioned by the discussers, the current research represents one of the few projects using a nontraditional internal bar placement (such as spirals near the supports). One of the purposes of these bar arrangements is to better understand the behavior of deep beams and applicability of strut-and-tie modeling in deep beams in which high concentrated compression and tension stresses occur.

EXPERIMENTAL PROCEDURE

The discusser raises the important point that determining the element forces in strut-and-tie models of indeterminate members is not always straightforward. The authors used the elastic finite element method (FEM) to provide guidance as to both the stress flow and the reaction forces (when needed). Additionally, the horizontal reactions in the pinned support were assumed to be zero for the purpose of design (verified by the elastic FEM). A plastic truss model was not used in this analysis.

With regard to stabilizer elements, a strut-and-tie model need not always be based on a stable truss—there are several cases in which a kinematic model can be satisfactorily used. This can be observed by the simple model used for a simply-supported, four-point loaded beam with direct struts from load points to supports.

The discussers next expressed interest with regard to the skin reinforcement used in each of the specimens. The 14-gauge welded wire reinforcement has wires with a cross-sectional area of 0.005 in.^{2} (3.2 mm^2) spaced at 1 in. (25.4 mm) in a mesh formation, which provided a reinforcement ratio of 0.0033 in both the horizontal and vertical directions. This reinforcement ratio is greater than the minimum reinforcement ratio and within typical values (between 0.003 and 0.006). As the failure occurred at the node regions in all the specimens, increased amounts of mesh reinforcement would not affect the overall strength of the specimens. If this transverse reinforcement ratio were to drop below the minimum amount required to hold the integrity of the struts, the mode and location of failure would have likely changed in addition to the ultimate failure load.

Regarding the reason neoprene bearings were chosen for all loading and support points, as there was no horizontal force applied to the beam, the authors chose to use all neoprene bearings for simplicity. The use of all neoprene bearings also better represented the assumptions made during the modeling of the specimens. No horizontal displacements were observed during testing.

The discusser finally asks whether any strains were measured in the main ties. Unfortunately, no tie strains were measures during testing. In retrospect, the tie strains would have been valuable in further investigations into estimating the deflections in such members. As the primary concern of the research was the ultimate strength of the specimens, and the predicted failure modes were controlled by the crushing of concrete, no steel strains were measured.

EXPERIMENTAL RESULTS AND DISCUSSION

The discussers reinforce the fact that the estimated design load (even when safety factors were not included) was conservative compared to the actual measured failure loads—something inherent to lower-bound theories. As mentioned, in statically indeterminate systems, there will be some extent of load redistribution that will ensue when local failures occur prior to a global failure. In this case, two local failures were required to cause a global failure. If the second local failure had a noticeably higher capacity than the first, then load distribution would result in increased capacity. If both local failure modes had similar capacities, then there would be no increased capacity due to load redistribution. The second local failure in Specimens A, B, and C was on the left side of the specimens (refer to Fig. 6). Specimen A had the least amount of steel crossing this failure crack, which may have been the cause of its lower strength.