

1 **Strength Predictions of Pile Caps by a Strut-and-Tie Model Approach**

2 JungWoong Park, Daniel Kuchma, and Rafael Souza

3 **JungWoong Park, Daniel Kuchma, and Rafael Souze**

4 Address:

5 Department of Civil and Environmental Engineering,

6 University of Illinois at Urbana-Champaign.

7 2114 Newmark Laboratory, 205 N. Mathews Ave., Urbana, IL, 61801, USA.

8
9 **Corresponding author: Daniel Kuchma**

10 Address:

11 Department of Civil and Environmental Engineering,

12 University of Illinois at Urbana-Champaign.

13 2106 Newmark Laboratory, 205 N. Mathews Ave., Urbana, IL, 61801, USA.

14 (217)-333-1571 (Phone)

15 (217)-333-9464 (Fax)

16 kuchma@uiuc.edu

1 **Abstract**

2 In this paper, a strut-and-tie model approach is presented for calculating the strength of
3 reinforced concrete pile caps. The proposed method employs constitutive laws for cracked
4 reinforced concrete and considers strain compatibility. This method is used to calculate the load
5 carrying capacity of 116 pile caps that have been tested to failure in structural research
6 laboratories. This method is illustrated to provide more accurate estimates of behavior and
7 capacity than the special provisions for slabs and footings of 1999 American Concrete Institute
8 (ACI) code, the pile cap provisions in the 2002 CRSI Design Handbook, and the strut-and-tie
9 model provisions in either 2005 ACI code or the 2004 Canadian Standards Association (CSA)
10 A23.3. The comparison shows that the proposed method consistently well predicts the strengths
11 of pile caps with shear span-to-depth ratios ranging from 0.49 to 1.8 and concrete strengths less
12 than 41 MPa. The proposed approach provides valuable insight into the design and behavior of
13 pile caps.

14 **Key words:** strut-and-tie model, pile caps, footings, failure strength, shear strength

15

16 **INTRODUCTION**

17 The traditional design procedure for pile caps is the same sectional approach as that typically
18 used for the design of two-way slabs and spread footings in which the depth is selected to
19 provide adequate shear strength from concrete alone and the required amount of longitudinal
20 reinforcement is calculated using the engineering beam theory assumption that plane sections
21 remain plane. However, and as illustrated by simple elastic analyses, pile caps are three-
22 dimensional D(Discontinuity) Regions in which there is a complex variation in straining not
23 adequately captured by sectional approaches. A new design procedure for all D-Regions,

1 including pile caps, has recently been introduced into North American design practice (Canadian
2 Standards Association (CSA) 1984, the American Association of State Highway and
3 Transportation Officials (AASHTO) 1994, American Concrete Institute (ACI) 2002). This
4 procedure is based on a strut-and-tie approach in which an idealized load resisting truss is
5 designed to carry the imposed loads through the discontinuity region to its supports. For the
6 typically stocky pile cap, such as the four-pile cap shown in Fig. 1, this consists of compressive
7 concrete struts that run between the column and the piles and steel reinforcement ties that extend
8 between piles.

9 The strut-and-tie approach is a conceptually simple and generally regarded as an appropriate
10 approach for the design of all D-Regions. To enable its use in practice, it was necessary to
11 develop specific rules for defining geometry and stress limits in struts and ties that have been
12 incorporated into codes of practice. These rules and limits were principally derived from tests on
13 planar structures and they are substantially different for the two predominant strut-and-tie design
14 provisions in North America, those being the “Design of Concrete Structures” by the Canadian
15 Standards Association (CSA Committee A23.3 2004) and Appendix A “Strut-and-Tie Models” of
16 the “Building Code Requirements for Structural Concrete” of the American Concrete Institute
17 (ACI Committee 318 2005). An evaluation of the applicability of these strut-and-tie provisions to
18 pile caps should be made using available experimental test data. In addition, it would be useful to
19 assess if the design of pile caps would benefit from any additional specific rules or guidelines in
20 order to ensure a safe and effective design.

21 This paper presents an examination of existing design methods for pile caps as well as a new
22 strut-and-tie approach for calculating the capacity of pile caps. This new approach utilizes
23 constitutive laws for cracked reinforced concrete and considers both strain compatibility and

1 equilibrium. To validate the proposed method, it is also used to calculate the strength of 116 pile
2 caps with concrete strengths less than 41 MPa. These strengths are also compared with those
3 calculated using the special provisions for slabs and footings of ACI 318-99 (ACI Committee
4 318 1999), CRSI Design Handbook 2002 (CRSI 2002), the strut-and-tie model provisions used
5 in ACI 318-05 (ACI Committee 318 2005) and the Canadian Standards Association (CSA
6 Committee A23.3 2004), and the strut-and-tie model approach presented by Adebar and Zhou
7 (1996).

8

9 **EXISTING PILE CAP DESIGN METHODS**

10 This section provides a brief discussion of the aforementioned provisions and guidelines that
11 are used in North American practice for the design of pile caps.

12 ACI 318-99 and CSRI Handbook suggest that pile caps be designed using the same
13 sectional design approaches as those for slender footings supported on soil. This requires a
14 design for flexure at the face of columns as well as one and two-way shear checks. The CSRI
15 Handbook provides an additional relationship for evaluating V_c when the shear span is less than
16 one-half the depth of the member, $w < d/2$, as presented in eq. [1] where c is the dimension of
17 a square column. These procedures are the most commonly used in North American design
18 practice.

19 [1]
$$V_c = \left(\frac{d}{w}\right)\left(1 + \frac{d}{c}\right)\left(0.33\sqrt{f'_c}\right)b_s d \quad (\text{mm, N})$$

20 where the shear section perimeter is $b_s = 4c$.

21 Appendix A of ACI 318-05 and the Canadian Standards Association provide provisions for
22 the design of all D(Discontinuity)-Regions in structural concrete, including pile caps. These

1 provisions include dimensioning rules as well as stress limits for evaluating the capacity of struts,
2 nodes, and the anchorage region of ties. They principally differ in the stress limits for struts. In
3 ACI 318-05, the compressive stress for the type of bottle shaped struts that occur in pile caps
4 would be $0.51f'_c$. The stress limit in struts by the CSA strut-and-tie provisions are a function of
5 the angle of the strut relative to the longitudinal axis, with the effect that the stress limit in 30, 45
6 and 60 degree struts with the assumption of tie strain $\epsilon_s = 0.002$ would be 0.31, 0.55, and
7 $0.73f'_c$, respectively. The strut-and-tie provisions in these code specifications have only had
8 limited use in design practice.

9 Based on an analytical and experimental study of compression struts confined by plain
10 concrete, Adebar and Zhou (1993) concluded that the design of pile caps should include a check
11 on bearing strength that is a function of the amount of confinement and the aspect ratio of the
12 diagonal struts. Adebar and Zhou (1996) provided the following equations for the maximum
13 allowable bearing stress in nodal zones:

14 [2; 3; 4]
$$f_b \leq 0.6f'_c + 6\alpha\beta\sqrt{f'_c}; \quad \alpha = \frac{1}{3}\left(\sqrt{A_2/A_1} - 1\right) \leq 1.0; \quad \beta = \frac{1}{3}\left(\frac{h_s}{b_s} - 1\right) \leq 1.0$$

15 The parameters α and β account for the confinement of the compression strut and the
16 geometry of the diagonal strut. The ratio A_2/A_1 in eq. [3] is identical to that used in the ACI
17 code for calculating the bearing strength. The ratio h_s/b_s is the aspect ratio (height-to-width) of
18 the strut. Adebar and Zhou suggested that the check described above is added to the traditional
19 section force approach for pile cap design.

20 The calculated strengths by these provisions and design guidelines are compared against the
21 test database following the presentation of the authors proposed strut-and-tie method and this test
22 database.

1

2 **A THREE-DIMENSIONAL STRUT-AND-TIE MODEL APPROACH**

3 To further evaluate the effectiveness of a strut-and-tie design approach for pile caps and to
4 identify means of improving design provisions, a methodology for evaluating the capacity of pile
5 caps was developed that considers strain compatibility and uses non-linear constitutive
6 relationship for evaluating the strength of struts. In this procedure, the three-dimensional strut-
7 and-tie model shown in Fig. 1 was used for the idealized load resisting truss in a four-pile cap.
8 This model is used for all pile caps examined in this paper. The shear span-to-depth ratio of most
9 test specimens selected in this study is less than one. Since the mode of failure is not known for
10 all test specimens, the proposed model considers the possibility of crushing of the compression
11 zone at the base of the column and yielding of the longitudinal reinforcement (ties). For all truss
12 models used in this study, the angle between longitudinal ties and diagonal struts is greater than
13 25 degrees; satisfying the ACI 318-05 limit. The details of the proposed strut-and-tie approach
14 are now presented.

15

16 **Effective depth of concrete strut**

17 The effective strut width is assumed based on the available concrete area and the anchorage
18 conditions of the strut. The effective area of diagonal strut at the top node is taken as

19 [5]
$$A_d = \frac{c}{\sqrt{2}} \left(\frac{c}{\sqrt{2}} \cos \theta_z + kd \sin \theta_z \right)$$

20 where c is the thickness of the square column and k is derived from the bending theory for a
21 single reinforced section as follows

22 [6]
$$k = \sqrt{(n\rho)^2 + 2n\rho} - n\rho$$

1 and where n is the ratio of steel to concrete elastic moduli with E_c taken as follows (Martinez
2 1982)

$$3 \quad [7] \quad E_c = \begin{cases} 4730\sqrt{f'_c} & \text{for } f'_c \leq 21 \text{ MPa} \\ 3320\sqrt{f'_c} + 6900 & \text{for } f'_c > 21 \text{ MPa} \end{cases}$$

4 The inclination angles between the diagonal struts and x-, y-, and z-axis are expressed as θ_x ,
5 θ_y , and θ_z respectively as shown in Fig. 1. These angles represent the direction cosines of a
6 diagonal strut. The effective area of a diagonal strut at the bottom node is taken as

$$7 \quad [8] \quad A_d = \frac{\pi}{4} d_p [d_p \cos \theta_z + 2(h-d) \sin \theta_z]$$

8 where d_p is pile diameter and h is overall height of the pile cap. The effective area of
9 diagonal strut is taken as the smaller of eqs. [5] and [8]. The effective depth of a horizontal strut
10 is taken as $h/4$ based on the suggestion of Paulay and Priestley (1992) on the depth of the
11 flexural compression zone of the elastic column as follows

$$12 \quad [9] \quad w_c = \left(0.25 + 0.85 \frac{N}{A_g f'_c} \right) h_c$$

13

14 **Force equilibrium**

15 The strut-and-tie model shown in Fig. 1 is statically determinate and thus member forces can
16 be calculated from the equilibrium equations only as given below:

$$17 \quad [10] \quad F_d = \frac{P}{4 \cos \theta_y}$$

$$18 \quad [11] \quad F_x = F_d \cos \theta_x$$

$$19 \quad [12] \quad F_y = F_d \cos \theta_y$$

1 where P is column load; F_d is the compressive forces in the diagonal strut; F_x and F_y are
 2 respectively the member forces in the x- and y-axis horizontal struts and ties. Since the strut-and-
 3 tie method is a full member design procedure; flexure and shear are not explicitly considered.

4

5 **Constitutive laws**

6 Cracked reinforced concrete can be treated as an orthotropic material with its principal axes
 7 corresponding to the directions of the principal average tensile and compressive strains. Cracked
 8 concrete subjected to high tensile strains in the direction normal to the compression is observed
 9 to be softer than concrete in a standard cylinder test (Hsu and Zhang 1997, Vecchio and Collins
 10 1982, 1986, 1993). This phenomenon of strength and stiffness reduction is commonly referred to
 11 as compression softening. Applying this softening effect to the strut-and-tie model, it is
 12 recognized that the tensile straining perpendicular to the compressive strut will reduce the
 13 capacity of the concrete strut to resist compressive stresses. Multiple compression softening
 14 models were used in this study to investigate the sensitively of the results to the selected model.
 15 All models were found to provide similarly good results as will be illustrated later in the paper.
 16 The compression softening model proposed by Hsu and Zhang (1997) was selected for the base
 17 comparisons and is now described, but it has been illustrated by the authors in a earlier paper
 18 (Park and Kuchma 2006) that different compression softening models can be similarly used. The
 19 stress of concrete strut is determined from the following equations proposed by Hsu and Zhang.

$$20 \quad [13] \quad \sigma_d = \xi f'_c \left[2 \left(\frac{\varepsilon_d}{\xi \varepsilon_0} \right) - \left(\frac{\varepsilon_d}{\xi \varepsilon_0} \right)^2 \right] \quad \text{for } \frac{\varepsilon_d}{\xi \varepsilon_0} \leq 1$$

$$21 \quad [14] \quad \sigma_d = \xi f'_c \left[1 - \left(\frac{\varepsilon_d / (\xi \varepsilon_0) - 1}{2/\xi - 1} \right)^2 \right] \quad \text{for } \frac{\varepsilon_d}{\xi \varepsilon_0} > 1$$

1 [15]
$$\xi = \frac{5.8}{\sqrt{f'_c}} \frac{1}{\sqrt{1+400\varepsilon_r}} \leq \frac{0.9}{\sqrt{1+400\varepsilon_r}}$$

2 where ε_0 is a concrete cylinder strain corresponding to the cylinder strength f'_c , which can be
 3 defined approximately as (Foster and Gilbert 1996)

4 [16]
$$\varepsilon_0 = 0.002 + 0.001 \left(\frac{f'_c - 20}{80} \right) \quad \text{for } 20 \leq f'_c \leq 100 \text{ MPa}$$

5 The response of the ties is based on the linear elastic perfectly plastic assumption.

6 [17]
$$F_{st} = E_s A_{st} \varepsilon_{st} \leq F_{st}$$

7 where A_{st} and F_{st} are the area and yielding force of horizontal steel tie in the x- or y-axes.

8 The proposed method considers a tension stiffening effect for evaluating the force and strain in
 9 steel ties. Vecchio and Collins (1986) suggested the following relationship for evaluating the
 10 average tensile stress in cracked concrete:

11 [18]
$$f_{ct} = \frac{f_{cr}}{1 + \sqrt{200\varepsilon_r}}$$

12 Taking f_{cr} as $0.33\sqrt{f'_c}$ and ε_r as 0.002, the tension force resisted by concrete tie is given by

13 [19]
$$F_{ct} = 0.20\sqrt{f'_c} A_{ct}$$

14 where A_{ct} is the effective area of concrete tie which is taken as

15 [20]
$$A_{ct} = \frac{d}{4} \left(\frac{l_e}{2} + \frac{d_p}{2} \right)$$

16 where l_e is the pile spacing.

17

18 **Compatibility relations**

19 The strain compatibility relation used in this study is the sum of normal strain in two
 20 perpendicular directions which is an invariant:

1 [21]
$$\varepsilon_h + \varepsilon_v = \varepsilon_r + \varepsilon_d$$

2 where ε_d is the compressive strain in a diagonal strut and ε_r is a tensile strain in the direction
3 perpendicular to the strut axis. Since horizontal and vertical web reinforcements were not
4 available from test data, ε_h and ε_v are conservatively taken as 0.002 in eq. [21].

5

6 **COMPARISON WITH TEST RESULTS**

7 **Existing test data**

8 Blevot and Fremy (1967) tested 59 four-pile caps. The majority of the four-pile caps were
9 approximately half-scale specimens, and eight of them were full-scale with 750-1000 mm overall
10 heights. Since one of main objectives of this work was to verify a truss analogy method, they
11 used different reinforcement details including no main reinforcement, and either uniformly
12 distributed or bunched reinforcement between piles. Clarke (1973) tested 15 square four-pile
13 caps with overall heights of 450 mm, all approximately half-scale. Two specimens had diagonal
14 main reinforcement, three had main reinforcement bunched over the piles, and the remaining ten
15 had uniformly distributed main reinforcement. The main variables in this study were pile spacing,
16 reinforcement layout, and anchorage type. He reported that the first cracks formed on the
17 centerlines of the vertical faces, and these cracks progressed rapidly upwards forming a
18 cruciform pattern, and finally each cap split into four blocks. Such observations point strongly to
19 a bending failure mode developing. However, though Clarke contended that the majority of the
20 caps failed in shear, the authors agree with Bloodworth, Jackson, and Lee (2003) that many of
21 these failure modes may be more accurately described as combined bending and shear failure.
22 Sabnis and Gogate (1984) tested nine small-scale four-pile caps with 152 mm overall heights, of
23 which one was unreinforced. They studied how the quantity of uniformly distributed longitudinal

1 reinforcement influences the shear capacity of deep pile caps. They reported that cracking of the
2 four outer faces was about the same in all the specimens and are indicative of combinations of
3 deep beam failure with very steep shear cracks and punching shear failures of slabs. They also
4 observed that some of this cracking may be prevented by the use of horizontal reinforcement on
5 the vertical faces of the caps; this reinforcement is only of secondary benefit and might not
6 substantially enhance the strength of the pile cap. Adebar, Kuchma, and Collins (1990) tested six
7 full-scale pile caps to study the performance of the strut-and-tie model for pile cap design. Four of
8 their tests were on diamond-shaped caps, one was on a cruciform-shaped cap, and one was on a
9 rectangular six-pile cap. The test results demonstrated that the strain distributions are highly
10 nonlinear both prior to cracking and after cracking. They reported that the failure occurs after a
11 compression strut split longitudinally due to the transverse tension caused by spreading of the
12 compressive stresses and that the maximum bearing stress is a good indicator of the likelihood of
13 a strut splitting failure. From the pile caps they tested, the maximum bearing stress at failure had
14 a lower limit of about $1.1f'_c$. They concluded that the strut-and-tie models accurately represent
15 the behavior of deep pile caps and correctly suggest that the load at which a lightly reinforced
16 pile cap fails in two-way shear depends on the quantity of longitudinal reinforcement. Suzuki,
17 Otsuki, and Tsubana (1998, 1999), Suzuki, Otsuki, and Tsuchiya (2000), and Suzuki and Otsuki
18 (2002) tested 94 four-pile caps with the reinforcement bunched over the piles or distributed in a
19 uniform grid. The main variables investigated in tests were the influence of edge distance, bar
20 arrangement, taper, and concrete strength on the failure mode and the ultimate strength. They
21 reported that it was experimentally observed that the ultimate strength of the pile caps with a
22 uniform grid arrangement was lower than that of pile caps with an equivalent amount of
23 reinforcement concentrate (bunched) between the pile bearings. Though pile caps may be

1 designed to any shape depending on the pile arrangement, rectangular four-pile caps previously
 2 tested were only chosen for examination in this study. Therefore, the 116 pile cap specimens
 3 tested by Clarke (1973), and Suzuki, Otsuki, and Tsubata (1998, 1999), Suzuki, Otsuki, and
 4 Tsuchiya (2000), Suzuki and Otsuki (2002), and Sabnis and Gogate (1984) were selected to
 5 validate the proposed method.

6

7 **Procedure for Evaluating the Capacity of Pile Caps**

8 The procedure for calculating the capacity of piles caps by the authors proposed method uses
 9 the compatibility, equilibrium, and constitutive relationships as described above and is as
 10 follows:

11 1. According to the member forces calculated from eq. [10] to eq. [12], ε_d and ε_r are found
 12 for P using eq. [13] and eq. [21], respectively. A concrete softening coefficient ξ is
 13 calculated from eq. [15] using ε_r .

14 2. The updated value of σ_d is calculated from eq. [13]. If the difference between the two σ_d
 15 values is larger than the defined tolerance, the steps are repeated until convergence has been
 16 achieved. Nominal strength by failure of diagonal strut can be estimated from

17 [22]
$$P_n = 4\xi f'_c A_d \cos \theta_z$$

18 3. The nominal strength by failure of horizontal concrete strut is taken by

19 [23]
$$P_n = 0.85 f'_c \frac{hc}{2} \frac{\cos \theta_z}{\cos \theta_x}$$

20 and, the nominal strength by tension failure mode can be expressed as

21 [24]
$$P_n = \left(2f_y A_s + 4F_{ct} \right) \frac{\cos \theta_z}{\cos \theta_x}$$

22 where f_y and A_s are the yield strength and cross-sectional area of the bottom longitudinal

1 reinforcement. The strength of the pile cap by a tension failure mode is the column load to cause
2 yielding of the reinforcement and fracture of a concrete tie.

3 4. The predicted strength by this method is the minimum value of the nominal strengths
4 computed from the different failure modes, which are crushing or splitting of the diagonal
5 concrete strut, crushing of the compression zone at the base of the column load, and yielding of
6 longitudinal reinforcement.

7

8 **Strength prediction**

9 The calculated strengths by the 6 methods (special provisions for slabs and footings of ACI
10 318-99 and in CRSI Design Handbook 2002, and the strut-and-tie methods in ACI 318-05, CSA
11 A23.3, by Adebar and Zhou, and by the Authors) are compared with the measured capacity of
12 the 116 selected pile caps test results. The details of the test specimens and strength ratios
13 (P_{test}/P_n) are presented for each of the 6 groups of test results in Tables 1-6, and collectively in
14 Table 8 and Figs. 2-3. In all figures, the shear span a is defined by the distance from pile
15 centre-line to column centre-line measured parallel to pile cap side. Table 7 shows the specimens
16 which were reported to have failed by shear. Some of specimens do not satisfy the code
17 minimum depth of 305 mm for footings on piles and the code minimum percentage of
18 longitudinal reinforcement. Especially, the overall height of the specimens of Sabnis and Gogate
19 (1984) is 152 mm which is about a half of code minimum footing depth, and 18 specimens of
20 Suzuki, Otsuki, and Tsubata (1999) are tapered pile caps. However, the comparative evaluation
21 still used this test data for the purpose of comparing the different design approaches. Tapered
22 pile caps can be designed using strut-and-tie model as long as the inclination of tapered pile cap
23 is small enough to include sufficient concrete area for the diagonal struts.

1 Fig. 2 presents the strength ratios (P_{test}/P_n) as a function of shear span-to-depth ratio for the
2 six aforementioned methods: (a) Special provisions for slabs and footings of ACI 318-99 Code;
3 (b) CRSI Design Handbook 2002; (c) Strut-and-tie model of ACI 318-05; (d) Strut-and-tie model
4 of CSA A23.3; (e) Strut-and-tie model approach of Adebar and Zhou; and (f) Proposed strut-
5 and-tie model approach by the authors. Based on these comparisons, the following initial
6 observations can be made. The special provisions in ACI 318-99 and the design formula of CRSI
7 Design Handbook 2002 lead to the most conservative estimates of strength with very reasonable
8 coefficients of variation for the range of tested pile caps. The strengths calculated by the strut-
9 and-tie provisions in Appendix A of ACI 318-05 and CSA A23.3 provide conservative estimates
10 of capacities and somewhat larger scatter of strength ratios. The methods presented by Adebar
11 and Zhou (1996) and the authors are less conservative, but still safe, with a scatter similar to that
12 by the ACI and CSRI special provisions for footings and slabs.

13 The above observations were referred to as initial observations for a more complete
14 examination of the behavior of the tested pile caps leads to a somewhat different assessment of
15 the accuracy and safety of these methods. The source of the conservatism of the first four
16 methods is that the calculated strengths, P_n , was usually controlled by the calculated flexural
17 capacity of the test structures. These calculated capacities have been observed to be unduly
18 conservative due to inaccuracies in the estimated flexural lever arm and ignoring tensile
19 contributions of the concrete. Therefore, in order to evaluate the shear provisions and the strut
20 and nodal zone stress limits of these methods, it is useful to examine the strength ratios for
21 members that did not fail by reinforcement yielding and in which the calculated strengths are not
22 limited by the calculated flexural capacity or strength of the tension ties.

23 Fig. 3 presents the strength ratios (P_{test}/P_n) as a function of shear span-to-depth ratio for the

1 six aforementioned methods for only those 33 pile caps that were reported by the authors to have
2 failed in shear and before reinforcement yielding and in which the nominal strength, P_n , is
3 controlled by the calculated shear strength or strength of struts and nodes. As shown in Fig. 3,
4 this leads to a very different impression of the accuracy and safety of these methods. The
5 calculated shear capacities by ACI 318-99 (Fig. 3a) and CSRI (Fig. 3b) were unconservative in
6 17 and 19 of the 33 cases, respectively. The strut and tie provisions by ACI 318-05 (Fig. 3c) and
7 the CSA A23.3 (Fig. 3d) were unconservative in 5 and 12 of the 33 cases, respectively. Thus, it
8 can be concluded that while these four methods are conservative due to their underprediction of
9 flexural and tie capacities, that the shear, concrete strut, and nodal zone capacities predicted by
10 these methods are unconservative.

11 Fig. 3(e) examines the accuracy of the strut-and-tie model approach proposed by Adebar and
12 Zhou (1996). The shear capacity predicted by this method is limited by the nodal zone bearing
13 stresses given by eq. [2], while the flexural capacity can be described by the column load that
14 would cause yielding of the steel tie of the strut-and-tie model. Adebar and Zhou (1996) assumed
15 that the lower nodes of strut-and-tie model were located at the center of the piles at the level of
16 the longitudinal reinforcement, while the upper nodal zones were assumed to be at the top
17 surface of the pile cap. This method does not overpredict any of the pile cap strengths and the
18 predictions are reasonably conservative as the strength of most pile caps was limited by the
19 conservative method for calculating the flexural capacity. However, the bearing capacity
20 requirement provides unconservative estimations of the strengths for many specimens which
21 were reported to have failed by shear as shown in Fig. 3(e). The shear span-to-depth ratios of
22 most test specimens reviewed in this study is less than one, and the majority of the specimens
23 may be more accurately described as combined bending and shear failure due to interpretation of

1 failure modes. The nodal zone bearing stress limit calculated in eq. [2] results in similar
2 maximum bearing strengths as calculated in the ACI Code in which the stress limit is
3 $\phi(0.85f'_c)\sqrt{A_2/A_1}$. Fig. 3(e) illustrates that the bearing strength limit of this method is not a good
4 indicator for pile cap strengths as has been reported by Cavers and Fenton (2004).

5 Figs. 2(f) and 3(f) examine the accuracy of the procedure developed by the authors. The
6 calculated capacities by the proposed method are both accurate and conservative with limited
7 scatter or trends for pile caps with shear span-to-depth ratios ranging from 0.49 to 1.8 and
8 concrete strength less than 41 MPa. The proposed method also provides reasonably conservative
9 strength predictions for all the specimens that were reported to have failed in shear.

10

11 **CONCLUSIONS**

12 In this paper, a three-dimensional strut-and-tie model approach has been presented for
13 calculating the load-carrying capacity of pile caps. The failure strength predictions for 116 tested
14 pile caps by this method are compared with those of six methods

15 1. The special provisions for slabs and footings of ACI 318-99 and the CSRI methods
16 provided the most conservative strength predictions. This conservatism is due to the particularly
17 low estimates of flexural capacity by these methods. If the shear provisions of these methods are
18 used to predict the capacity of those members that are reported to have failed in shear, then these
19 shear provisions are found to be quite unconservative; the capacity of more than one-half of the
20 tested shear-critical pile caps are over predicted.

21 2. The strut-and-tie model approaches in Appendix A of ACI 318-05 and the CSA A23.3 did
22 not overpredict the measured strengths of any of the pile caps. However, the provisions of these
23 methods for calculating the strength of struts and nodes by these methods were found to be

1 somewhat unconservative for those members that did not fail by reinforcement yielding.

2 3. The strut-and-tie approach by Adebar and Zhou did not overpredict the strength of any of
3 the pile caps that failed by yielding of the longitudinal reinforcement and these strength
4 predictions were reasonably accurate. However, this approach provided somewhat
5 unconservative estimations of the shear strengths for many of the test specimens that were
6 reported to have failed by shear.

7 4. The calculated capacities by the proposed method were both accurate and conservative with
8 little scatter or trends for tested pile caps with shear span-to-depth ratios ranging from 0.49 to 1.8
9 and concrete strength less than 41 MPa. The success of the proposed method indicates that a
10 strut-and-tie design philosophy is appropriate for the design of pile caps.

1 **List of symbols:**

- 2 a, d, h the distance from pile centre-line to column center-line measured parallel to pile cap
3 side, effective depth, overall height
- 4 A_d, A_{ct} effective areas of diagonal strut and concrete tie
- 5 A_s cross-sectional area of main reinforcement
- 6 b_o perimeter of critical section
- 7 c, d_p, l_e column size, pile diameter, pile spacing
- 8 f'_c compressive strength of concrete cylinder
- 9 f_{cr} concrete tensile strength
- 10 f_{ct} tensile stress of concrete tie
- 11 f_{cu} effective strength of concrete strut
- 12 f_y yield strength of reinforcement
- 13 F_{ct} nominal strength of concrete tie
- 14 F_d, F_x, F_y the forces of diagonal, x, and y-directional members
- 15 w distance between column face and center line of pile
- 16 $\theta_x, \theta_y, \theta_z$ inclination angle between diagonal strut and x, y, and z-axis
- 17 θ_s inclination angel between diagonal strut and steel tie
- 18 w_c, w_d effective width of horizontal strut and diagonal strut
- 19 σ_d compressive stress of concrete strut
- 20 ε_0 strain at peak stress of standard cylinder
- 21 ε_s tensile strain of steel tie
- 22 $\varepsilon_h, \varepsilon_v$ strain of horizontal direction and vertical direction

- 1 ε_d compressive strain of diagonal strut
- 2 ε_r tensile strain of the direction perpendicular to diagonal strut

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- 1 **Table captions:**
- 2 **Table 1** – Test data of Clarke (1973)
- 3 **Table 2** – Test data of Suzuki, Otsuki, and Tsubata (1998)
- 4 **Table 3** – Test data of Suzuki, Otsuki, and Tsubata (1999)
- 5 **Table 4** – Test data of Suzuki, Otsuki, and Tsuchiya (2000)
- 6 **Table 5** – Test data of Suzuki, and Otsuki (2002)
- 7 **Table 6** – Test data of Sabnis and Gogate (1984)
- 8 **Table 7** – Test specimens reported to have failed by shear
- 9 **Table 8** – Ratio of measured to predicted strength

- 1 **Figure captions:**
- 2 **Fig. 1** – A strut-and-tie model for pile caps
- 3 **Fig. 2** – Ratio of measured to predicted strength with respect to shear span-depth ratio: (a)
- 4 Special provisions for slabs and footings of ACI 318-99; (b) CRSI Design Handbook 2002; (c)
- 5 Strut-and-tie model of ACI 318-05; (d) Strut-and-tie model of CSA A23.3; (e) Strut-and-tie
- 6 model approach of Adebar and Zhou; (f) Proposed strut-and-tie model approach
- 7 **Fig. 3** – Ratio of measured to calculated strengths by shear failure mode with respect to shear
- 8 span-depth ratio: (a) Special provisions for slabs and footings of ACI 318-99; (b) CRSI Design
- 9 Handbook 2002; (c) Strut-and-tie model of ACI 318-05; (d) Strut-and-tie model of CSA A23.3;
- 10 (e) Strut-and-tie model approach of Adebar and Zhou; (f) Proposed strut-and-tie model approach

Table 1 – Test data of Clarke (1973)

pile cap	f'_c (MPa)	cap size (mm×mm)	l_e (mm)	(a)	bar arrangement
A1	21.3	950×950	600	10	grid
A2	27.2	950×950	600	10	bunched
A4	21.4	950×950	600	10	grid
A5	26.6	950×950	600	10	bunched
A7	24.2	950×950	600	10	grid
A8	27.2	950×950	600	10	bunched
A9	26.6	950×950	600	10	grid
A10	18.8	950×950	600	10	grid
A11	18.0	950×950	600	10	grid
A12	25.3	950×950	600	10	grid
B1	26.7	750×750	400	8	grid
B2	24.5	750×750	400	10	grid
B3	35.0	750×750	400	6	grid

Note: (a) number of D10 bars at both of x and y direction; pile spacing l_e ; yield strength of reinforcement $f_y=410$ MPa, overall height $h=450$ mm, effective depth $d=405$ mm, column width $c=200$ mm, pile diameter $d_p=200$ mm for all specimens

Table 2 – Test data of Suzuki, Otsuki, and Tsubata (1998)

pile cap	f'_c (MPa)	cap size (mm×mm)	l_e (mm)	h (mm)	d (mm)	c (mm)	(a)	f_y (MPa)		bar arrangement
								x-dir.	y-dir.	
BP-20-1	21.3	900×900	540	200	150	300	8	413	413	grid
BP-20-2	20.4	900×900	540	200	150	300	8	413	413	grid
BPC-20-1	21.9	900×900	540	200	150	300	8	413	413	bunched
BPC-20-2	19.9	900×900	540	200	150	300	8	413	413	bunched
BP-25-1	22.6	900×900	540	250	200	300	10	413	413	grid
BP-25-2	21.5	900×900	540	250	200	300	10	413	413	grid
BPC-25-1	18.9	900×900	540	250	200	300	10	413	413	bunched
BPC-25-2	22.0	900×900	540	250	200	300	10	413	413	bunched
BP-20-30-1	29.1	800×800	500	200	150	300	6	405	405	grid
BP-20-30-2	29.8	800×800	500	200	150	300	6	405	405	grid
BPC-20-30-1	29.8	800×800	500	200	150	300	6	405	405	bunched
BPC-20-30-2	29.8	800×800	500	200	150	300	6	405	405	bunched
BP-30-30-1	27.3	800×800	500	300	250	300	8	405	405	grid
BP-30-30-2	28.5	800×800	500	300	250	300	8	405	405	grid
BPC-30-30-1	28.9	800×800	500	300	250	300	8	405	405	bunched
BPC-30-30-2	30.9	800×800	500	300	250	300	8	405	405	bunched
BP-30-25-1	30.9	800×800	500	300	250	250	8	405	405	grid
BP-30-25-2	26.3	800×800	500	300	250	250	8	405	405	grid
BPC-30-25-1	29.1	800×800	500	300	250	250	8	405	405	bunched
BPC-30-25-2	29.2	800×800	500	300	250	250	8	405	405	bunched
BDA-70-90-1	29.1	700×900	500	300	250	250	8	356	345	grid
BDA-70-90-2	30.2	700×900	500	300	250	250	8	356	345	grid
BDA-80-90-1	29.1	800×900	500	300	250	250	8	356	345	grid
BDA-80-90-2	29.3	800×900	500	300	250	250	8	356	345	grid
BDA-90-90-1	29.5	900×900	500	300	250	250	8	356	345	grid
BDA-90-90-2	31.5	900×900	500	300	250	250	8	356	345	grid
BDA-100-90-1	29.7	1000×900	500	300	250	250	8	356	345	grid
BDA-100-90-2	31.3	1000×900	500	300	250	250	8	356	345	grid

Note: (a) number of D10 bars at both of x and y direction; pile diameter $d_p=150$ mm for all specimens

Table 3 – Test data of Suzuki, Otsuki, and Tsubata (1999)

pile cap	f'_c (MPa)	l_e (mm)	h (mm)	d (mm)	(a)	(b)
TDL1-1	30.9	600	350	300	4	356
TDL1-2	28.2	600	350	300	4	356
TDL2-1	28.6	600	350	300	6	356
TDL2-2	28.8	600	350	300	6	356
TDL3-1	29.6	600	350	300	8	356
TDL3-2	29.3	600	350	300	8	356
TDS1-1	25.6	450	350	300	6	356
TDS1-2	27.0	450	350	300	6	356
TDS2-1	27.2	450	350	300	8	356
TDS2-2	27.3	450	350	300	8	356
TDS3-1	28.0	450	350	300	11	356
TDS3-2	28.1	450	350	300	11	356
TDM1-1	27.5	500	300	250	4	383
TDM1-2	26.3	500	300	250	4	383
TDM2-1	29.6	500	300	250	6	383
TDM2-2	27.6	500	300	250	6	383
TDM3-1	27.0	500	300	250	10	370
TDM3-2	28.0	500	300	250	10	370

Note: (a) number of D10 bars at both of x and y direction; (b) yield strength of reinforcement at both of x and y direction in MPa; pile cap size 900×900 mm, column width $c=250$ mm, pile diameter $d_p=150$ mm, grid type of bar arrangement for all specimens

Table 4 – Test data of Suzuki, Otsuki, and Tsuchiya (2000)

pile cap	f'_c (MPa)	cap size (mm×mm)	h (mm)	d (mm)	c (mm)	(a)	(b)
BDA-20-25-70-1	26.1	700×700	200	150	250	4	358
BDA-20-25-70-2	26.1	700×700	200	150	250	4	358
BDA-20-25-80-1	25.4	800×800	200	150	250	4	358
BDA-20-25-80-2	25.4	800×800	200	150	250	4	358
BDA-20-25-90-1	25.8	900×900	200	150	250	4	358
BDA-20-25-90-2	25.8	900×900	200	150	250	4	358
BDA-30-20-70-1	25.2	700×700	300	250	200	6	358
BDA-30-20-70-2	24.6	700×700	300	250	200	6	358
BDA-30-20-80-1	25.2	800×800	300	250	200	6	358
BDA-30-20-80-2	26.6	800×800	300	250	200	6	358
BDA-30-20-90-1	26.0	900×900	300	250	200	6	358
BDA-30-20-90-2	26.1	900×900	300	250	200	6	358
BDA-30-25-70-1	28.8	700×700	300	250	250	6	383
BDA-30-25-70-2	26.5	700×700	300	250	250	6	383
BDA-30-25-80-1	29.4	800×800	300	250	250	6	383
BDA-30-25-80-2	27.8	800×800	300	250	250	6	383
BDA-30-25-90-1	29.0	900×900	300	250	250	6	383
BDA-30-25-90-2	26.8	900×900	300	250	250	6	383
BDA-30-30-70-1	26.8	700×700	300	250	300	6	358
BDA-30-30-70-2	25.9	700×700	300	250	300	6	358
BDA-30-30-80-1	27.4	800×800	300	250	300	6	358
BDA-30-30-80-2	27.4	800×800	300	250	300	6	358
BDA-30-30-90-1	27.2	900×900	300	250	300	6	358
BDA-30-30-90-2	24.5	900×900	300	250	300	6	358
BDA-40-25-70-1	25.9	700×700	400	350	250	8	358
BDA-40-25-70-2	24.8	700×700	400	350	250	8	358
BDA-40-25-80-1	26.5	800×800	400	350	250	8	358
BDA-40-25-80-2	25.5	800×800	400	350	250	8	358
BDA-40-25-90-1	25.7	900×900	400	350	250	8	358
BDA-40-25-90-2	26.0	900×900	400	350	250	8	358

Note: (a) number of D10 bars at both of x and y direction; (b) yield strength of reinforcement at both of x and y direction in MPa; pile spacing $l_e=450$ mm, pile diameter $d_p=150$ mm, grid type of bar arrangement for all specimens

Table 5 – Test data of Suzuki, and Otsuki (2002)

pile cap	f'_c (MPa)	c (mm)	anchorage
BPL-35-30-1	24.1	300	180-deg. hook
BPL-35-30-2	25.6	300	180-deg. hook
BPB-35-30-1	23.7	300	bent-up
BPB-35-30-2	23.5	300	bent-up
BPH-35-30-1	31.5	300	180-deg. hook
BPH-35-30-2	32.7	300	180-deg. hook
BPL-35-25-1	27.1	250	180-deg. hook
BPL-35-25-2	25.6	250	180-deg. hook
BPB-35-25-1	23.2	250	bent-up
BPB-35-25-2	23.7	250	bent-up
BPH-35-25-1	36.6	250	180-deg. hook
BPH-35-25-2	37.9	250	180-deg. hook
BPL-35-20-1	22.5	200	180-deg. hook
BPL-35-20-2	21.5	200	180-deg. hook
BPB-35-20-1	20.4	200	bent-up
BPB-35-20-2	20.2	200	bent-up
BPH-35-20-1	31.4	200	180-deg. hook
BPH-35-20-2	30.8	200	180-deg. hook

Note: 9-D10 bars at both of x and y direction; yield strength of reinforcement $f_y=353$ MPa; pile cap size 800×800 mm, pile spacing $l_e=500$ mm, overall height $h=350$ mm, effective depth $d=300$ mm, pile diameter $d_p=150$ mm, grid type of bar arrangement for all specimens

Table 6 – Test data of Sabnis and Gogate (1984)

pile cap	f'_c (MPa)	d (mm)	(a)	(b)
SS1	31.3	111	0.0021	499
SS2	31.3	112	0.0014	662
SS3	31.3	111	0.00177	886
SS4	31.3	112	0.0026	482
SS5	41.0	109	0.0054	498
SS6	41.0	109	0.0079	499
SG1	17.9	152	0	-
SG2	17.9	117	0.0055	414
SG3	17.9	117	0.0133	414

Note: (a) reinforcement ratio at both of x and y direction; (b) yield strength of reinforcement at both of x and y direction in MPa; pile cap size 330×330 mm, pile spacing $l_e=203$ mm, overall height $h=152$ mm, column diameter $c=76$ mm, pile diameter $d_p=76$ mm, grid type of bar arrangement for all specimens

Table 7 – Test specimens reported to have failed by shear

Author	pile cap specimens
Clarke (1973)	A1, A2, A4, A5, A7, A8, A9, A10
Suzuki, Otsuki, and Tsubata (1998)	BP-25-1, BP-25-2, BP-30-30-1, BP-30-25-2
Suzuki, Otsuki, and Tsubata (1999)	BDA-40-25-70-1
Suzuki, Otsuki, and Tsuchiya (2000)	TDM3-1, TDM3-2
Suzuki, and Otsuki (2002)	BPL-35-30-1, BPL-35-30-2, BPH-35-30-1, BPL-35-25-2, BPH-35-25-1, BPH-35-25-2, BPL-35-20-1, BPL-35-20-2, BPH-35-20-1, BPH-35-20-2
Sabnis and Gogate (1984)	SS1, SS2, SS3, SS4, SS5, SS6, SG2, SG3

Table 8 – Ratio of measured to predicted strength

specimen	P_{test} (kN)	P_{test} / P_n						specimen	P_{test} (kN)	P_{test} / P_n					
		(a)	(b)	(c)	(d)	(e)	(f)			(a)	(b)	(c)	(d)	(e)	(f)
BP-20-1	519	2.08	2.08	1.69	1.80	1.43	1.51	BDA-20-25-70-1	294	2.22	2.22	1.93	2.03	1.57	1.46
BP-20-2	480	1.93	1.93	1.57	1.67	1.32	1.45	BDA-20-25-70-2	304	2.29	2.29	1.99	2.10	1.62	1.51
BPC-20-1	519	2.08	2.08	1.69	1.80	1.43	1.48	BDA-20-25-80-1	304	2.29	2.29	1.99	2.10	1.62	1.51
BPC-20-2	529	2.13	2.13	1.73	1.84	1.46	1.64	BDA-20-25-80-2	304	2.29	2.29	1.99	2.10	1.62	1.51
BP-25-1	735	1.76	1.76	1.52	1.46	1.22	1.51	BDA-20-25-90-1	333	2.50	2.50	2.18	2.30	1.77	1.65
BP-25-2	755	1.81	1.81	1.64	1.51	1.25	1.63	BDA-20-25-90-2	333	2.50	2.50	2.18	2.30	1.77	1.65
BPC-25-1	818	1.98	1.98	2.02	1.64	1.35	2.01	BDA-30-20-70-1	534	1.61	1.61	1.40	1.50	1.23	1.12
BPC-25-2	813	1.95	1.95	1.73	1.62	1.35	1.72	BDA-30-20-70-2	549	1.65	1.65	1.44	1.54	1.26	1.16
BP-20-30-1	485	2.40	2.40	1.93	2.02	1.63	1.62	BDA-30-20-80-1	568	1.71	1.71	1.49	1.60	1.30	1.19
BP-20-30-2	480	2.38	2.38	1.91	2.00	1.62	1.60	BDA-30-20-80-2	564	1.69	1.69	1.48	1.58	1.29	1.18
BPC-20-30-1	500	2.48	2.48	1.99	2.08	1.68	1.67	BDA-30-20-90-1	583	1.75	1.75	1.53	1.64	1.34	1.22
BPC-20-30-2	495	2.45	2.45	1.97	2.06	1.67	1.65	BDA-30-20-90-2	588	1.76	1.76	1.54	1.65	1.35	1.23
BP-30-30-1	916	2.03	2.03	1.52	1.58	1.39	1.34	BDA-30-25-70-1	662	1.86	1.86	1.47	1.54	1.32	1.21
BP-30-30-2	907	2.01	2.01	1.50	1.57	1.37	1.32	BDA-30-25-70-2	676	1.90	1.90	1.50	1.57	1.35	1.24
BPC-30-30-1	1039	2.30	2.30	1.72	1.79	1.57	1.51	BDA-30-25-80-1	696	1.95	1.95	1.54	1.62	1.39	1.27
BPC-30-30-2	1029	2.28	2.28	1.71	1.77	1.56	1.49	BDA-30-25-80-2	725	2.03	2.03	1.61	1.69	1.44	1.33
BP-30-25-1	794	1.76	1.76	1.44	1.51	1.29	1.23	BDA-30-25-90-1	764	2.14	2.14	1.69	1.78	1.52	1.39
BP-30-25-2	725	1.61	1.61	1.32	1.39	1.18	1.14	BDA-30-25-90-2	764	2.14	2.14	1.69	1.78	1.52	1.40
BPC-30-25-1	853	1.89	1.89	1.55	1.62	1.38	1.33	BDA-30-30-70-1	769	2.31	2.31	1.64	1.72	1.51	1.38
BPC-30-25-2	872	1.93	1.93	1.58	1.66	1.42	1.36	BDA-30-30-70-2	730	2.20	2.20	1.56	1.63	1.44	1.31
BDA-70-90-1	784	1.97	1.97	1.62	1.70	1.45	1.36	BDA-30-30-80-1	828	2.48	2.48	1.77	1.85	1.63	1.48
BDA-70-90-2	755	1.89	1.89	1.56	1.63	1.39	1.30	BDA-30-30-80-2	809	2.43	2.43	1.73	1.81	1.59	1.44
BDA-80-90-1	858	2.15	2.15	1.77	1.86	1.58	1.49	BDA-30-30-90-1	843	2.52	2.52	1.80	1.88	1.66	1.51
BDA-80-90-2	853	2.14	2.14	1.76	1.85	1.58	1.48	BDA-30-30-90-2	813	2.44	2.44	1.74	1.81	1.60	1.47
BDA-90-90-1	853	2.14	2.14	1.76	1.84	1.58	1.48	BDA-40-25-70-1	1019	1.64	1.64	1.24	1.29	1.16	1.12
BDA-90-90-2	921	2.31	2.31	1.90	1.99	1.70	1.59	BDA-40-25-70-2	1068	1.72	1.72	1.30	1.35	1.22	1.23
BDA-100-90-1	911	2.28	2.28	1.88	1.97	1.68	1.58	BDA-40-25-80-1	1117	1.79	1.79	1.36	1.41	1.28	1.20
BDA-100-90-2	931	2.33	2.33	1.92	2.01	1.72	1.60	BDA-40-25-80-2	1117	1.80	1.80	1.36	1.41	1.28	1.25
A1	1110	1.44	1.44	1.73	1.53	1.17	1.10	BDA-40-25-90-1	1176	1.89	1.89	1.43	1.49	1.34	1.31
A2	1420	1.83	1.83	1.73	1.74	1.50	1.33	BDA-40-25-90-2	1181	1.89	1.89	1.43	1.49	1.35	1.30
A4	1230	1.59	1.59	1.91	1.69	1.30	1.22	TDL1-1	392	1.94	1.94	1.68	1.74	1.53	1.06
A5	1400	1.80	1.80	1.75	1.72	1.48	1.31	TDL1-2	392	1.95	1.95	1.68	1.74	1.53	1.08
A7	1640	2.12	2.12	2.25	2.03	1.73	1.55	TDL2-1	519	1.72	1.72	1.48	1.54	1.35	1.10
A8	1510	1.95	1.95	1.84	1.85	1.59	1.41	TDL2-2	472	1.57	1.57	1.35	1.40	1.23	1.00
A9	1450	1.87	1.87	1.81	1.78	1.53	1.36	TDL3-1	608	1.52	1.52	1.30	1.35	1.19	1.04
A10	1520	1.97	1.97	2.68	2.38	1.60	1.71	TDL3-2	627	1.57	1.57	1.34	1.39	1.22	1.07
A11	1640	2.13	2.13	3.02	2.68	1.73	1.92	TDS1-1	921	2.30	2.30	1.77	1.85	1.64	1.44
A12	1640	2.12	2.12	2.15	2.02	1.73	1.55	TDS1-2	833	2.08	2.08	1.60	1.67	1.48	1.29
B1	2080	2.23	2.23	2.29	2.29	1.65	1.79	TDS2-1	1005	1.89	1.89	1.45	1.52	1.34	1.24
B2	1900	1.64	1.64	2.28	2.28	1.20	1.78	TDS2-2	1054	1.98	1.98	1.52	1.59	1.41	1.30
B3	1770	2.52	2.52	1.97	2.06	1.87	1.50	TDS3-1	1299	1.78	1.78	1.40	1.44	1.26	1.42
BPL-35-30-1	960	1.81	1.81	1.32	1.38	1.24	1.26	TDS3-2	1303	1.79	1.79	1.40	1.45	1.26	1.42
BPL-35-30-2	941	1.77	1.77	1.30	1.35	1.21	1.18	TDM1-1	490	2.27	2.27	1.88	1.97	1.68	1.37
BPB-35-30-1	1029	1.94	1.94	1.42	1.48	1.32	1.38	TDM1-2	461	2.13	2.13	1.77	1.85	1.58	1.30
BPB-35-30-2	1103	2.08	2.08	1.52	1.59	1.42	1.49	TDM2-1	657	2.03	2.03	1.68	1.76	1.50	1.35
BPH-35-30-1	980	1.83	1.83	1.35	1.40	1.26	1.16	TDM2-2	657	2.04	2.04	1.68	1.76	1.50	1.36
BPH-35-30-2	1088	2.04	2.04	1.50	1.55	1.40	1.28	TDM3-1	1245	1.53	1.44	1.72	1.21	0.99	1.72
BPL-35-25-1	902	1.69	1.69	1.36	1.42	1.24	1.16	TDM3-2	1210	1.46	1.38	1.61	1.17	0.97	1.63
BPL-35-25-2	872	1.64	1.64	1.31	1.38	1.20	1.13	SS1	250	3.04	3.04	2.76	2.96	2.48	2.31
BPB-35-25-1	911	1.72	1.72	1.37	1.45	1.26	1.30	SS2	245	3.40	3.40	3.07	3.23	2.78	2.52
BPB-35-25-2	921	1.73	1.73	1.38	1.46	1.27	1.29	SS3	248	2.04	2.04	2.71	2.04	1.65	1.72
BPH-35-25-1	882	1.65	1.65	1.33	1.38	1.22	1.10	SS4	226	2.32	2.32	2.42	2.29	1.89	1.81
BPH-35-25-2	951	1.78	1.78	1.43	1.49	1.31	1.15	SS5	264	1.61	1.61	2.21	1.66	1.09	1.43
BPL-35-20-1	755	1.42	1.42	1.24	1.33	1.11	1.15	SS6	280	1.71	1.71	2.34	1.76	1.16	1.52
BPL-35-20-2	735	1.39	1.39	1.21	1.30	1.08	1.17	SG1	50	-	-	-	-	-	1.53
BPB-35-20-1	755	1.43	1.43	1.31	1.34	1.11	1.27	SG2	173	1.43	1.43	3.11	2.49	1.20	1.97
BPB-35-20-2	804	1.52	1.52	1.41	1.43	1.18	1.37	SG3	177	1.46	1.46	3.20	2.55	1.23	2.01
BPH-35-20-1	813	1.52	1.52	1.33	1.41	1.20	1.10	Average		1.97	1.96	1.73	1.74	1.44	1.41
BPH-35-20-2	794	1.49	1.49	1.30	1.38	1.17	1.08	Coefficient of Variation		0.17	0.17	0.24	0.20	0.18	0.18

Note: P_{test} = measured failure load; (a) Special provisions for slabs and footings of ACI 318-99; (b) CRSI Design Handbook 2002; (c) Strut-and-tie model of ACI 318-05; (d) Strut-and-tie model of CSA A23.3; (e) Strut-and-tie model approach of Adebar and Zhou; (f) Proposed strut-and-tie model approach

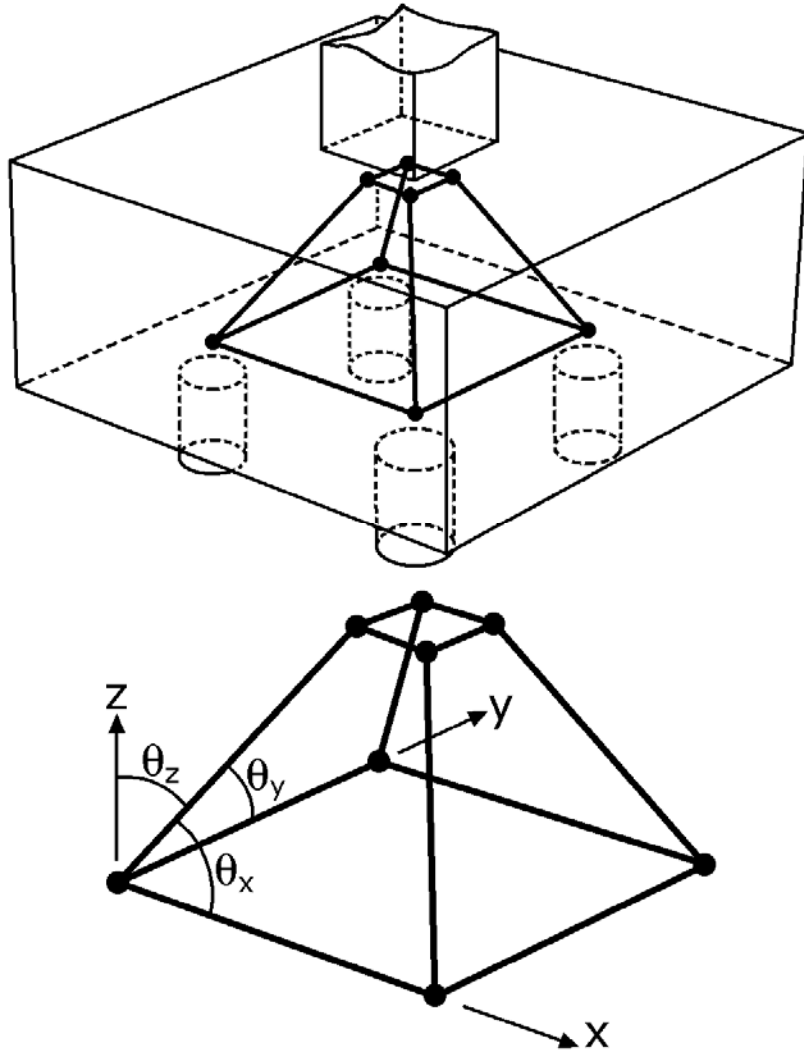
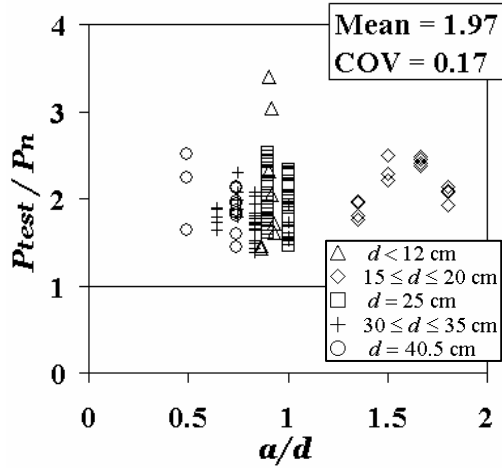
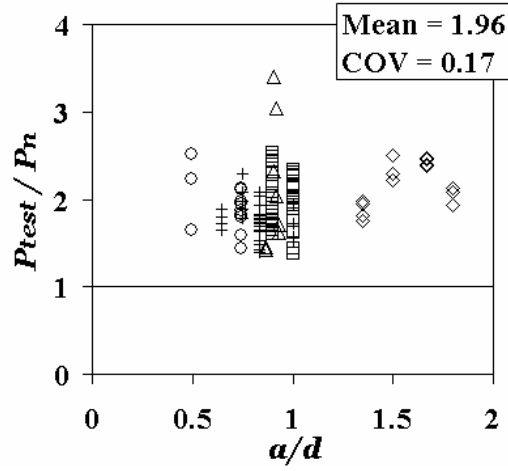


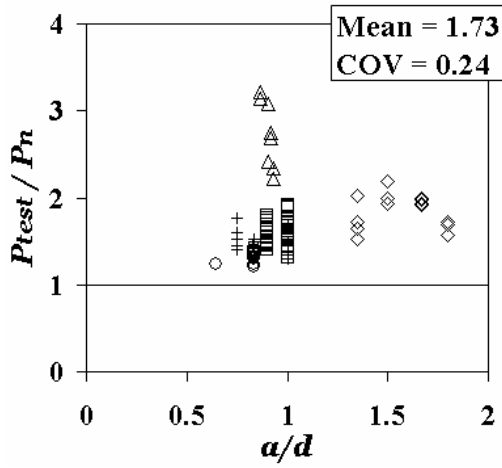
Fig. 1 – A strut-and-tie model for pile caps



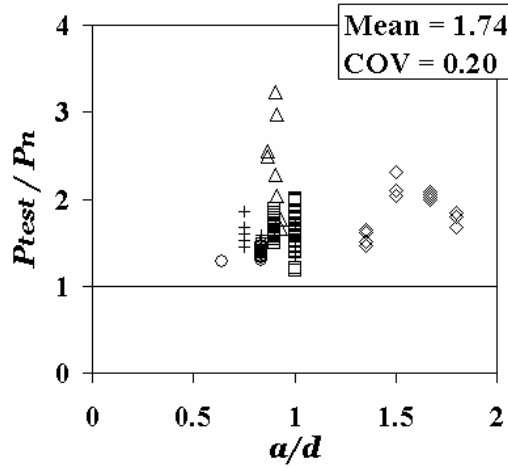
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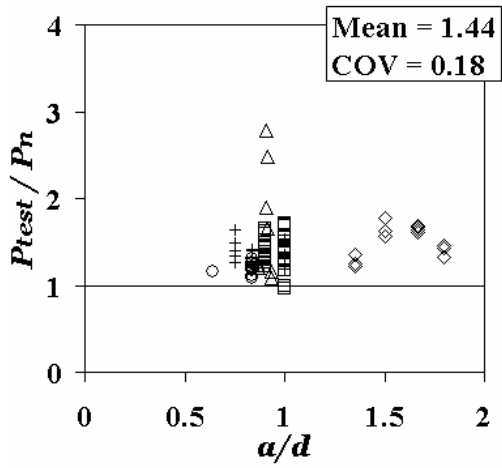
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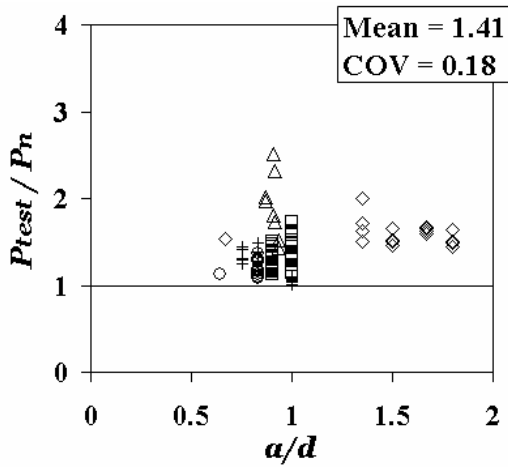
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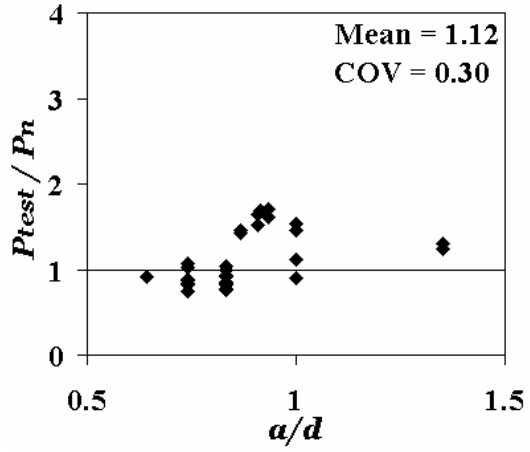


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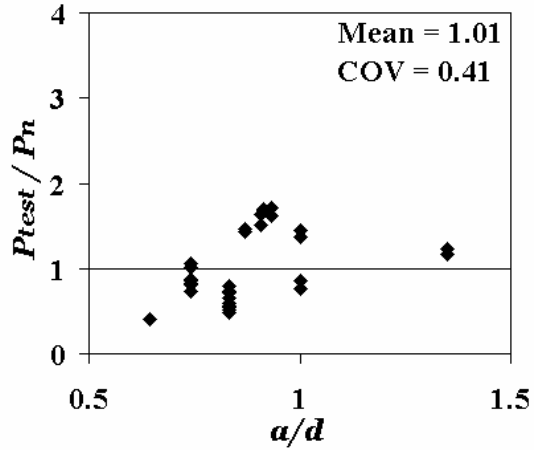


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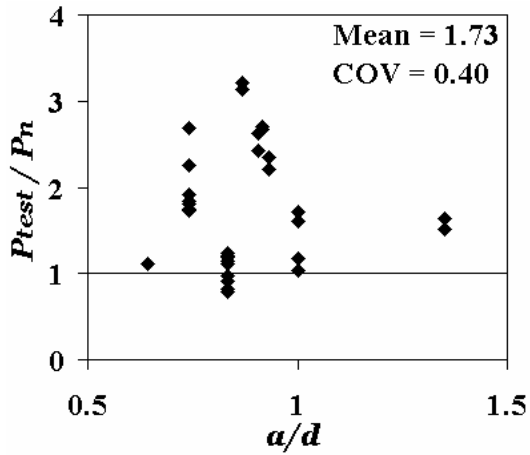
Fig. 2 – Ratio of measured to predicted strength with respect to shear span-depth ratio: (a) Special provisions for slabs and footings of ACI 318-99; (b) CRSI Design Handbook 2002; (c) Strut-and-tie model of ACI 318-05; (d) Strut-and-tie model of CSA A23.3; (e) Strut-and-tie model approach of Adebar and Zhou; (f) Proposed strut-and-tie model approach



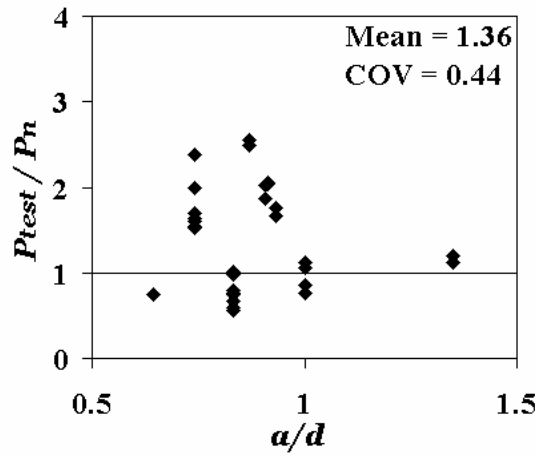
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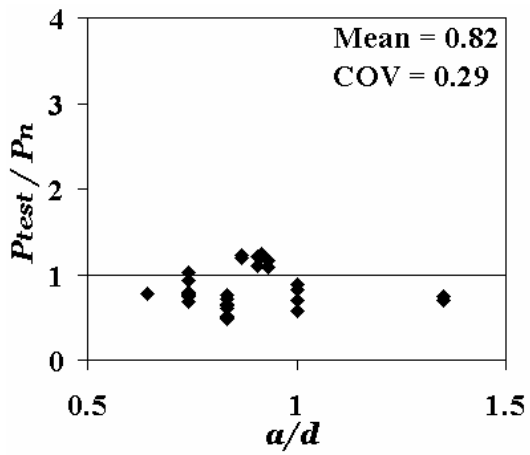
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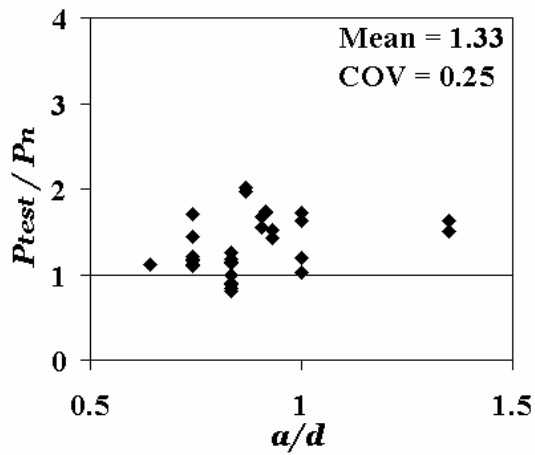
(c)



(d)



(e)



(f)

Fig. 3 – Ratio of measured to calculated shear strengths for the specimens failed by shear

with respect to shear span-depth ratio: (a) Special provisions for slabs and footings of ACI 318-99; (b) CRSI Design Handbook 2002; (c) Strut-and-tie model of ACI 318-05; (d) Strut-and-tie model of CSA A23.3; (e) Strut-and-tie model approach of Adebar and Zhou; (f) Proposed strut-and-tie model approach