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Adaptable Strut-and-Tie Model for Design and Verification of Four-Pile Caps

by Rafael Souza, Daniel Kuchma, JungWoong Park, and Túlio Bittencourt

A large percentage of pile caps support only one column, and the pile caps in turn are supported by only a few piles. These are typically short and deep members with overall span-depth ratios of less than 1.5. Codes of practice do not provide uniform treatment for the design of these types of pile caps. These members have traditionally been designed as beams spanning between piles with the depth selected to avoid shear failures and the amount of longitudinal reinforcement selected to provide sufficient flexural capacity as calculated by the engineering beam theory. More recently, the strut-and-tie method has been used for the design of pile caps (disturbed or D-region) in which the load path is envisaged to be a three-dimensional truss, with compressive forces being supported by concrete compressive struts between the column and piles and tensile forces being carried by reinforcing steel located between piles. Both of these models have not provided uniform factors of safety against failure or been able to predict whether failure will occur by flexure (ductile mode) or shear (fragile mode). In this paper, an analytical model based on the strut-and-tie approach is presented. The proposed model has been calibrated using an extensive experimental database of pile caps subjected to compression and evaluated analytically for more complex loading conditions. It has been proven to be applicable across a broad range of test data and can predict the failures modes, cracking, yielding, and failure loads of four-pile caps with reasonable accuracy.

Keywords: design; flexural strength; pile caps; shear strength; strut-and-tie model.

INTRODUCTION

Pile caps are used to transfer the load from one or more columns to a group of piles. Despite being a very common and important element in construction, there is no generally accepted procedure for the design of pile caps; many empirical detailing rules are followed in practice, but these approaches vary significantly. The main reason for these disparities is that most codes do not provide a design methodology that provides a clear understanding of the strength and behavior of this important structural element.

Some designers and codes[1-4] assume that a pile cap acts as a beam spanning between piles, select the depth to provide adequate shear strength, and determine the required longitudinal reinforcement based on the engineering beam theory in which a linear distribution of strain is assumed over the depth of a member. The traditional ACI Building Code[1,5-7] design procedure for pile caps uses the same sectional approach used for footings supported on soil and for two-way slabs. Other design provisions[1,4,8] use a strut-and-tie procedure in which an internal load-resisting truss, with compressive forces being taken by concrete compressive struts and tensile forces carried by steel reinforcement ties, is assumed to transfer the loads from the column to the supporting piles. A general strut-and-tie design procedure for all discontinuity (D)-regions was introduced into U.S. design practice with Appendix A in ACI 318-02.[7]

Linear and nonlinear analyses illustrate that pile caps behave as three-dimensional elements in which there is a complex variation in straining over the dimensions of the D-region and in which compressive struts develop between columns and piles. For this reason, design procedures for pile caps should not be based on a sectional design method and many tests have illustrated the imperfection of this approach.[9-13] Of particular concern was that many pile caps that were designed to fail in flexure have been reported to fail in the brittle mode of shear.[14-19] The strut-and-tie method provides a more suitable procedure for proportioning the dimensions and selecting the reinforcement for pile caps.

To more accurately predict the behavior of pile caps, not only for the case of bunched square or grid reinforcement but also for a combination of both layouts, this paper presents an approach for developing a three-dimensional strut-and-tie model. The proposed model, which has been calibrated using extensive experimental data,[14-19] is able to predict, with fairly good accuracy, the failure mode, as well as the cracking, yielding, and failure loads of four-pile caps.

RESEARCH SIGNIFICANCE

The presented research contributes to the development of design recommendations for pile caps under complex loading, providing useful guidance for determining crack, yield, and failure load of four-pile caps. The appropriateness of the proposed strut-and-tie model is based on experimental data collected from literature, and the applied methodology can be easily extended for caps supported on a larger number of piles. The use of the proposed model can provide more economical, safe, and rational models for the design of pile caps than the application of a sectional design method.

EXPERIMENTAL DATA FOR FOUR-PILE CAPS

There are limited experimental test data on the performance of pile caps and most of this is for the behavior of two- or four-pile caps. Unfortunately, a significant portion of these results are not useful for evaluating code provisions, as the reinforcement patterns used in the test specimens are not consistent with design methodologies. For this reason, only test results from four-pile caps containing concentrated reinforcement (bunched squares) or mesh pattern (grid) are used in the development and validation of the procedure that is presented in this paper. A brief summary of these test results is presented.

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A comprehensive series of tests were carried out by Blévot and Frémy,[14] who tested 51 four-pile caps at approximately half-scale and eight four-pile caps at full-scale. The main objective of these tests was to check the validity of different strut-and-tie models as well as to compare the performance of pile caps containing different patterns of longitudinal reinforcement. The tests results showed that the use of bunched square layouts resulted in a 20% higher failure load than in members with the same quantity of reinforcement distributed in a grid pattern. It was observed in these tests, however, that the use of only a bunched square layout of reinforcement resulted in poor crack control, and for this reason, the use of complementary grid reinforcement was recommended by these authors.

According to Blevót and Frémy,[14] interpretation of results regarding pile caps is very difficult because punching shear can occur at failure. These authors concluded that while it is possible to separate bending and shear behavior in beams, it is not possible in pile caps, because an increase of longitudinal reinforcement produced a significant increase in punching strength. This observation is supported by other collected pile cap test data,[14-19] as shown in Fig. 1.

Clarke[15] tested 15 four-pile caps at half-scale with different patterns of longitudinal reinforcement. All of these caps were conservatively designed for flexure. Clarke observed that the sectional approach for calculating shear capacity was unsafe. Only four specimens failed by flexure, whereas the remainder of the specimens failed by shear after yielding of the longitudinal reinforcement.

The main conclusion of Clarke[15] was that CEB[20] and CP110[21] exaggerate the importance of the effective depth for calculating shear strength. For this reason, two new approaches were proposed and later incorporated into the BS 8110[3] and BS 5400[2] codes. The tests results also illustrated that using a bunched square reinforcement pattern resulted in failure loads that were 25% higher than the failure loads measured when the same amount of reinforcement was arranged in a grid pattern, thus confirming the conclusions obtained previously by Blévot and Frémy.[15]

Sabnis and Gogate[22] tested nine small 1/5-scale four-pile caps, varying the grid reinforcement ratio from 0.21% to 1.33%. The objective of these tests was to determine if the amount of longitudinal reinforcement had an influence on shear strength, and the main conclusion was that the amount of reinforcement over 0.2% had little or no influence on capacity. This conclusion is not supported by a broader range of test data.[14-19]

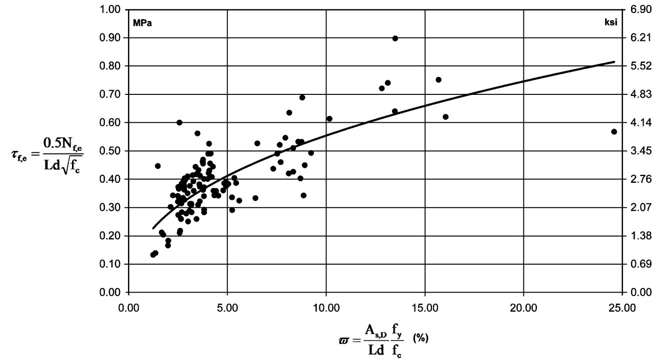

Fig. 1—Effect of mechanical reinforcement ratio on shear strength of tested pile caps.[14-19] (Note: 1 MPa = 6.895 ksi.)

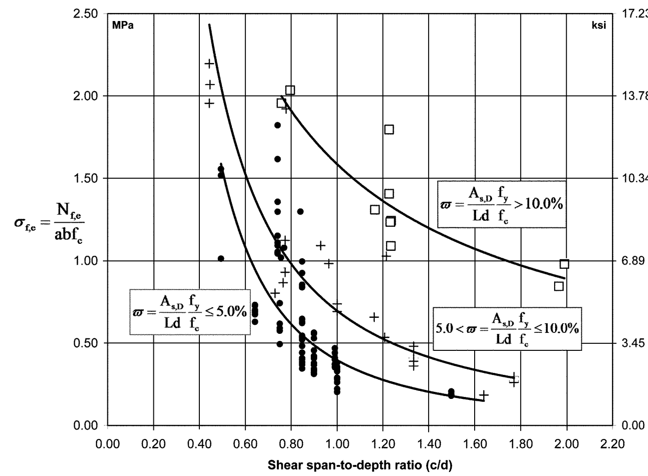

Fig. 2—Effect of shear span-depth ratio on maximum normal stress acting on columns of tested pile caps.[14-19] (Note: 1 MPa = 6.895 ksi.)

Adebar et al.[9] tested five four-pile caps with complex geometries and concluded that the strut-and-tie model can well predict the general behavior of this complex three-dimensional element. Failures usually occurred after yielding of the longitudinal reinforcement and with splitting of the diagonal compressive struts that extended from the column to the piles. This splitting is also characteristic of reported shear failures in deep beams. To prevent compressive strut splitting failures, these authors suggested to limit the maximum bearing stress at the top of a pile cap to $1.0f_c$. Despite that this suggestion is simple to use, this proposal was observed to not be valid for all member ranges, as shown in Fig. 2. As can be seen, the maximum normal stress at failure acting in the columns of tested pile caps[14-19] is dependent on the shear span-depth ratio and the mechanical reinforcement ratio.

An important conclusion made by Adebar et al.[9] was that ACI 318-83[5] fails to capture the trend of experimental test results. They suggest that the ACI 318-83 exaggerates the importance of effective depth and that the strength of a deep pile cap is better enhanced by increasing the bearing area of concentrated loads than by increasing the depth of this pile cap.

Adebar and Zhou,[23] based on an analytical and experimental study of compression struts confined by concrete, proposed a simple method to verify shear strength, in which the maximum bearing stress is considered a better indicator than shear stress on any prescribed critical section. In further

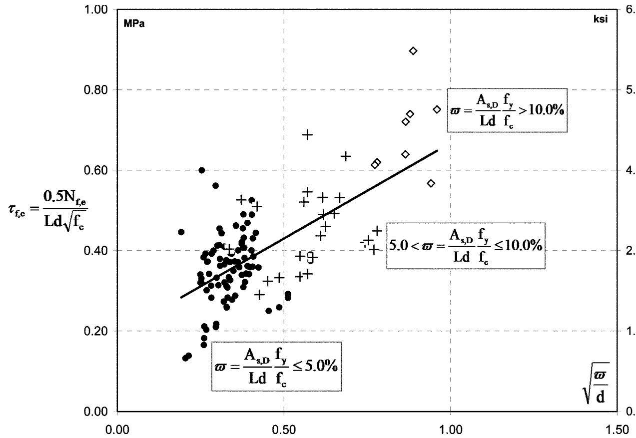

Fig. 3—Effect of reinforcement ratio and depth on shear strength of tested pile caps.[14-19] (Note: 1 MPa = 6.895 ksi.)

work, Adebar and Zhou[10] compared their previously proposed model with the results of 48 pile caps. It was concluded that the one-way shear design provisions of ACI 318-83[5] are excessively conservative and that the traditional flexure design procedures for beams and two-way slabs are unconservative for pile caps. To overcome these problems, Adebar and Zhou[10] suggested that pile caps designed using a strut-and-tie method be checked using an additional and indirect verification for shear. This proposal is based on the premise presented by Schlaich et al.[25]—that an entire D-region designed using a strut-and-tie model can be considered safe if the maximum bearing stress is maintained below a certain limit.

Suzuki et al.[18] tested 28 four-pile caps in which the longitudinal bar layouts and edge distances (the shortest distance from the peripheral of the footing slab to the center of the pile) were varied. Most specimens failed by shear after yielding of the longitudinal reinforcement and only four pile caps failed by shear without yielding of the longitudinal reinforcement. It was observed that bunched square layouts led to higher strengths and that edge distance affected the failure load. To increase strength and deformation capacity even after reinforcement yields, the edge distance was recommended to be approximately 1.5 times the pile diameter.

Suzuki et al.[17] tested 18 four-pile caps having top inclined slabs (tapered footings) and demonstrated that cracking load tends to decrease as the reinforcement ratio increases. In these tests, most pile caps failed in shear after yielding of the longitudinal reinforcement, and just two pile caps failed by shear before reinforcement yielding.

Suzuki et al.[19] tested 34 pile caps with reinforcement provided in a grid layout. An important objective of the research was to evaluate the influence of edge distance between the piles and the cap on strength and behavior. The tests have shown that cracking load and flexural capacity decreases even if the reinforcement amount in the slab is the same when the edge distance is shortened.

Suzuki and Otsuki[16] tested 18 four-pile caps with reinforcement distributed in a grid. The test parameters included concrete strength and type of anchorage. The edge distance was kept equal to the pile diameter and it was possible to conclude that concrete strength has no influence on failure mode and ultimate strength. In most specimens, the failure mode was due to shear that occurred before reinforcement yielding. While all pile caps were predicted to fail by flexure, 10 of the specimens did not fail in this mode, and the authors concluded that it was due to the influence of shortened edge distances on shear failure. Although the authors did not make a specific reference to anchorage length conditions, it seems clear that short edge distances will directly affect the development of the longitudinal reinforcement. The same anchorage problem seems to occur in the experimental data obtained by Suzuki et al.[19]

While it was not a primary objective of this paper to identify trends in the test data, the influence of a few key variables on measured failure stresses was examined to better understand the characteristics of the test data. As previously mentioned and illustrated in Fig. 1, the normalized beam shear stress at failure, $\tau_{f,e}$, increases with the mechanical reinforcement ratio ϖ, as observed in beam test data for members without shear reinforcement. The strength of pile caps is in part controlled by the coupled interaction between mechanical reinforcement ratio and shear span-depth ratio. Figure 2 illustrates that the normalized stress in the column at failure increases with mechanical reinforcement ratio and decreases with the span-depth ratio. Figure 3 illustrates that the normalized shear stress at pile cap failure increases with the square root of the ratio of mechanical reinforcement to depth, $\sqrt{\varpi/d}$. The trends of the data indicate that the sectional design method becomes more appropriate for shear span-depth ratios (c/d) greater than 1.5, whereas members are controlled by splitting failures when c/d is less than 0.5. The most appropriate range of application is in the dominate range for pile caps designs in which the range of shear span-depth ratios is between 0.5 and 1.5. This model is now presented.

STRUT-AND-TIE MODEL TO PREDICT BEHAVIOR OF FOUR-PILE CAPS

Souza et al.[24] proposed an adaptable tridimensional truss model for the generic case of axial compression N_k and biaxial flexure (M_{kx}, M_{ky}) being imposed on a pile cap that is supported by four piles, as shown in Fig. 4. The boundary conditions of the proposed space truss are defined in such a way that rigid-body translation and rotation are prohibited and a statically determinate structure is obtained. This model, which was evaluated using nonlinear analysis, enables direct consideration of these three actions and avoids the need for inaccurate simplifications that are often applied in practice. A simplified version of this model is calibrated using the measured response of four-pile caps that supported a square column subjected to axial load, by setting $e_{x,k} = e_{y,k} = M_{x,k} = M_{y,k} = 0$. In this way, one can illustrate that the reactions on the piles, the internal angles, and the forces in the struts and ties can be calculated as follows

$$R_{A,k} = R_{B,k} = R_{C,k} = R_{D,k} = \frac{N_k}{4} \quad (1)$$

$$\text{tg}\theta_A = \text{tg}\theta_B = \text{tg}\theta_C = \text{tg}\theta_D = \frac{d}{0.70711e} \quad (2)$$

$$C_{A,k} = C_{B,k} = C_{C,k} = C_{D,k} = \frac{N_k}{4\sin\theta_A} \quad (3)$$

$$T_{AB,k} = T_{AC,k} = T_{CD,k} = T_{BD,k} = \frac{-0.125N_k e}{d} \quad (4)$$

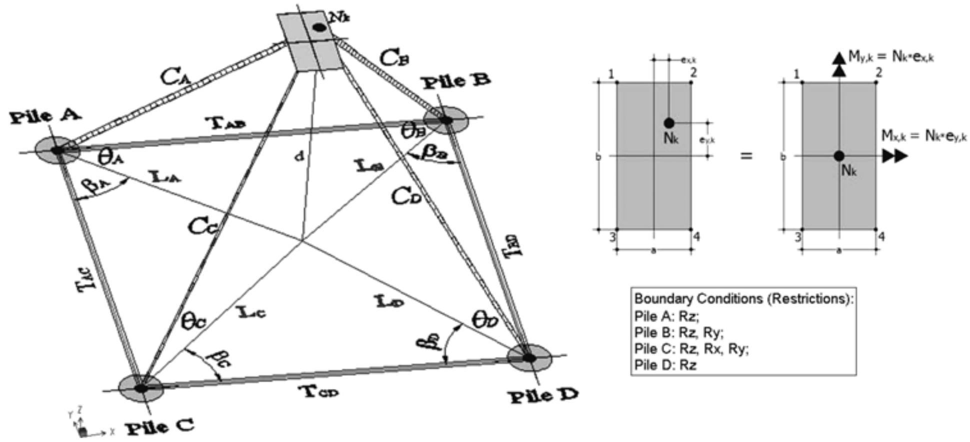

Fig. 4—Proposed strut-and-tie model for four-pile caps supporting rectangular columns that impose axial compression and biaxial flexure.[24]

The axial load corresponding to yielding of this reinforcement can be determined by rearranging Eq. (4) to form Eq. (5). In this equation, the parameter ϕ_y was introduced to provide a calibration with experimental test data

$$N_{y,a} = -\frac{A_{sT} f_y d}{0.125e} = -\frac{8\phi_y A_{sT} f_y d}{e} \qquad (5)$$

Equation (5) is only applicable to the case when a square grid of concentrated tension ties is provided. To predict when yielding of reinforcement will occur for both bunched and grid reinforcement, Eq. (6) may be applied in which A_{sD} is the total reinforcement in the considered direction. In the same equation, d is the depth and e is the distance between piles

$$N_{y,a} = -\frac{4\phi_y A_{sD} f_y d}{e} \qquad (6)$$

To predict the failure modes and failure loads for four-pile caps, the simple suggestion proposed by Siao[24] was adopted. In this formulation, shear failure of pile caps is assumed to be related to the splitting of compressive struts, and for that reason, shear failure is dependent on the column/pile dimensions as well as the tensile strength of concrete. Equation (7) shows the proposal by Siao,[25] while Eq. (9) is a modified version of this equation that accounts for the dimensions of the square column and the tensile strength of concrete given by Eq. (8) as calculated using the CEB-FIP Model Code[27] recommendation

$$N_{fs,a} = -4f_t(a + b)d \qquad (7)$$

$$f_t = 0.26f_c^{2/3} \text{ (MPa)} \qquad (8)$$

$$N_{fs,a} = -4f_t(b + b)d = -2.08bdf_c^{2/3}, f_c \text{ in MPa} \qquad (9)$$

Assuming that the four-pile cap will fail by shear (strut splitting) or by flexure, a single criterion for predicting failure load and failure mode can be proposed in which the flexural failure capacity is evaluated by Eq. (6) and shear (strut splitting) capacity is equated by Eq. (9), as shown in Eq. (10). In this last equation, if $N_{ff,a} < N_{fs,a}$, then $N_{f,a} = N_{ff,a}$ and failure is due to flexure, and if $N_{ff,a} > N_{fs,a}$ then $N_{f,a} = N_{fs,a}$ and failure is due to shear

$$N_{f,a} \le \begin{cases} N_{ff,a} = \dfrac{-4\phi_f A_{sD} f_y d}{e} \rightarrow \text{flexure failure mode} \\ N_{fs,a} = -2.08bdf_c^{2/3} \rightarrow \text{shear failure mode} \end{cases} \qquad (10)$$

Finally, an expression for the axial load that was experimentally measured to produce the first cracks in four-pile caps is given by Eq. (11), in which ϕ_c is a calibration coefficient from the test data, e is the distance between centers of piles, d is the depth, and L is the pile cap width

$$N_{c,a} = -\frac{\phi_c L d f_c^{2/3}}{e}, f_c \text{ in MPa} \qquad (11)$$

APPLICATION OF PROPOSED PREDICTION MODEL TO EXPERIMENTAL DATA

The proposed strut-and-tie model was applied to experimental data obtained by Blévot and Frémy,[14] Clarke,[15] Suzuki et al.,[17-19] and Suzuki and Otsuki[16] for members with a shear span-depth ratio ranging from $0.44 \le c/d \le 1.99$. To provide an evaluation of the method for current design practice, just-tested pile caps with bunched square, grid longitudinal, or both layouts were analyzed. The behavior of pile caps with less common reinforcement layouts, principally those with diagonal reinforcement, was not considered in the analysis.

The coefficients ϕ_c, ϕ_y, and ϕ_f are, respectively, coefficients of calibration for cracking, yielding, and failure loads. They were derived from experimental test data to provide the lowest possible coefficients of variation. For evaluating failure mode and failure load, the results from 129 specimens were used. For cracking and yielding, only 67 and 69 were used due to availability of information in experimental datasets.

Tables 1 to 6 present the geometric details of the tested four-pile caps, their material properties, as well as their measured strengths. These tables also show the predicted cracking, yielding, and failure loads (shear or flexure) of the proposed model calibrated with calibration coefficients of $\phi_c = 0.101$, $\phi_y = 1.88$, and $\phi_f = 2.05$.

Table 1—Comparison between proposed analytical model and experimental data from Blévot and Frémy[14]

Geometrical properties, material properties, and experimental results										Predictions of proposed model				Comparisons		
L, m	h, m	e, m	$a = b$, m	p,* m	A_{sD}, cm^2	d, m	f_c, MPa	f_y, MPa	$\|N_{f,e}\|$, kN	$\|N_{c,a}\|$, kN	$\|N_{y,a}\|$, kN	$\|N_{ff,a}\|$, kN	$\|N_{fs,a}\|$, kN	$N_{f,e}/N_{f,a}$	EFL	AFL
0.60	0.30	0.42	0.15	0.14	4.04	0.25	29.10	439.70	850	341	795.15	867.05	737.94	1.15	s	s
0.60	0.30	0.42	0.15	0.14	6.28	0.28	32.60	283.50	747.5	409	886.19	966.32	885.14	0.84	s	s
0.60	0.20	0.42	0.15	0.14	4.04	0.18	32.10	469.00	475	263	613.37	668.83	569.76	0.83	s	s
0.60	0.30	0.42	0.15	0.14	7.66	0.27	26.60	494.50	1150	347	1831.16	1996.75	750.65	1.53	s	s
0.60	0.20	0.42	0.15	0.14	7.66	0.17	29.15	509.35	815	232	1187.58	1294.97	502.38	1.62	s	s
0.60	0.20	0.42	0.15	0.14	4.03	0.17	33.90	459.50	408	263	576.91	629.08	568.64	0.72	s	s
0.60	0.30	0.42	0.15	0.14	6.30	0.27	30.75	342.30	650	385	1050.23	1145.20	832.95	0.78	s	s
0.60	0.30	0.42	0.15	0.14	4.03	0.27	21.00	325.30	510	300	640.80	698.74	648.33	0.79	s	s
0.60	0.14	0.42	0.15	0.14	6.28	0.11	13.15	498.00	250	86	598.04	652.12	185.64	1.35	s	s
0.60	0.14	0.42	0.15	0.14	12.32	0.11	13.15	461.00	290	85	1072.83	1169.85	183.38	1.58	s	s
0.60	0.20	0.42	0.15	0.14	6.28	0.18	22.05	512.00	650	205	1037.99	1131.85	442.34	1.47	s	s
0.60	0.20	0.42	0.15	0.14	10.32	0.17	30.60	476.00	850	241	1499.61	1635.22	520.43	1.63	s	s
0.60	0.30	0.42	0.15	0.14	9.04	0.26	18.40	517.50	842.5	265	2211.32	2411.28	574.08	1.47	s	s
0.60	0.30	0.42	0.15	0.14	16.08	0.28	18.40	468.00	810	279	3736.37	4074.24	603.00	1.34	s	s
0.60	0.50	0.42	0.15	0.14	9.04	0.47	27.27	459.00	1200	619	3520.76	3839.13	1339.57	0.90	s	s
0.60	0.50	0.42	0.15	0.14	16.08	0.47	40.81	467.00	1900	805	6330.06	6902.46	1741.15	1.09	s	s
0.60	0.50	0.42	0.15	0.14	18.09	0.47	34.40	450.25	1700	723	6908.19	7532.87	1563.26	1.09	s	s
0.60	0.25	0.42	0.15	0.14	9.04	0.23	34.60	446.00	850	347	1632.92	1780.58	749.37	1.13	s	s
0.60	0.25	0.42	0.15	0.14	9.04	0.22	33.93	453.30	750	330	1598.75	1743.32	712.53	1.05	s	s
0.60	0.30	0.42	0.15	0.14	9.04	0.27	26.88	311.00	562.5	352	1367.18	1490.81	760.39	0.74	s	s
0.60	0.30	0.42	0.15	0.14	9.04	0.27	19.48	311.00	492.5	284	1367.18	1490.81	613.49	0.80	s	s
0.60	0.30	0.42	0.15	0.14	6.28	0.29	30.86	444.70	557.5	409	1439.08	1569.21	883.44	0.63	s	s
0.60	0.30	0.42	0.15	0.14	6.28	0.27	30.00	440.70	585	383	1360.73	1483.77	827.18	0.71	s	s
0.60	0.20	0.42	0.15	0.14	9.04	0.17	20.78	318.70	840	187	883.13	962.98	403.73	2.08	s	s
0.60	0.20	0.42	0.15	0.14	9.04	0.17	21.88	318.70	692.5	193	883.13	962.98	417.85	1.66	s	s
0.60	0.20	0.42	0.15	0.14	6.28	0.17	32.43	435.70	750	254	847.05	923.65	548.59	1.37	s	s
0.60	0.20	0.42	0.15	0.14	6.28	0.17	26.10	431.70	640	216	826.17	900.88	467.24	1.37	s	s
—	0.75	1.20	0.50	0.35	78.42	0.67	37.25	277.75	7000	—	9076.93	9897.72	7713.87	0.91	s	s
—	0.75	1.20	0.50	0.35	48.24	0.68	40.80	479.55	6700	—	9785.47	10,670.32	8319.76	0.81	s	s
—	1.00	1.20	0.50	0.35	60.85	0.92	34.15	274.96	6500	—	9593.74	10,461.26	10,016.47	0.65	s	s
—	1.00	1.20	0.50	0.35	39.41	0.92	49.30	453.33	9000	—	10,328.18	11,262.11	12,899.25	0.79	s	f

*Used square piles.

Note: EFL = experimental failure load; AFL = analytical failure load; s = shear failure; f = flexural failure; 1 m = 39.37 in.; 1 cm^2 = 0.155 in.^{2}; 1 kN = 0.225 kip; and 1 MPa = 6.895 ksi.

Table 2—Comparison between proposed analytical model and experimental data from Clarke[15]

Geometrical properties, material properties, and experimental results										Predictions of proposed model				Comparisons		
L, m	h, m	e, m	$a = b$, m	p, m	A_{sD}, cm^2	d, m	f_c, MPa	f_y, MPa	$\|N_{f,e}\|$, kN	$\|N_{c,a}\|$, kN	$\|N_{y,a}\|$, kN	$\|N_{ff,a}\|$, kN	$\|N_{fs,a}\|$, kN	$N_{f,e}/N_{f,a}$	EFL	AFL
0.95	0.45	0.6	0.2	0.2	7.85	0.40	26.6	410	1110	570	1614.36	1760.34	1482.77	0.75	s	s
0.95	0.45	0.6	0.2	0.2	7.85	0.40	34	410	1420	671	1614.36	1760.34	1746.38	0.81	s	s
0.95	0.45	0.6	0.2	0.2	7.85	0.40	26.7	410	1230	571	1614.36	1760.34	1486.49	0.83	s	s
0.95	0.45	0.6	0.2	0.2	7.85	0.40	33.2	410	1400	661	1614.36	1760.34	1718.88	0.81	s	s
0.95	0.45	0.6	0.2	0.2	7.85	0.40	30.2	410	1640	620	1614.36	1760.34	1613.71	1.02	s	s
0.95	0.45	0.6	0.2	0.2	7.85	0.40	34	410	1510	671	1614.36	1760.34	1746.38	0.86	s	s
0.95	0.45	0.6	0.2	0.2	7.85	0.40	33.2	410	1450	661	1614.36	1760.34	1718.88	0.84	s	s
0.95	0.45	0.6	0.2	0.2	7.85	0.40	23.5	410	1520	525	1614.36	1760.34	1365.21	1.11	s	s
0.95	0.45	0.6	0.2	0.2	7.85	0.40	22.5	410	1640	510	1614.36	1760.34	1326.20	1.24	f	s
0.95	0.45	0.6	0.2	0.2	7.85	0.40	31.6	410	1640	639	1614.36	1760.34	1663.20	0.99	f	s
0.75	0.45	0.4	0.2	0.2	6.28	0.40	33.4	410	2080	786	1937.24	2112.41	1725.78	1.21	s	s
0.75	0.45	0.4	0.2	0.2	7.85	0.40	30.8	410	1870	744	2420.31	2639.17	1635.01	1.14	s	s
0.75	0.45	0.4	0.2	0.2	4.71	0.40	43.7	410	1770	940	1452.93	1584.31	2064.47	1.11	f	f

Note: EFM = experimental failure load; AFL = analytical failure load; s = shear failure; f = flexural failure; 1 m = 39.37 in.; 1 cm^2 = 0.155 in.^{2}; 1 kN = 0.225 kip; and 1 MPa = 6.895 ksi.

Table 3—Comparison between proposed analytical model and experimental data from Suzuki et al.[18]

Geometrical properties, material properties, and experimental results												Predictions of proposed model				Comparisons				
L, m	h, m	e, m	$a = b$, m	p, m	A_{sD}, cm^2	d, m	f_c, MPa	f_y, MPa	$\|N_{c,e}\|$, kN	$\|N_{y,e}\|$, kN	$\|N_{f,e}\|$, kN	$\|N_{c,a}\|$, kN	$\|N_{y,a}\|$, kN	$\|N_{ff,a}\|$, kN	$\|N_{fs,a}\|$, kN	$N_{c,e}/N_{c,a}$	$N_{y,e}/N_{y,a}$	$N_{f,e}/N_{f,a}$	EFL	AFL
0.9	0.2	0.54	0.3	0.15	5.67	0.15	21.9	413	176	510	519	193	478.15	521.39	716.18	0.91	1.06	0.99	f+p	f
0.9	0.2	0.54	0.3	0.15	5.67	0.15	19.9	413	167	519	529	181	478.15	521.39	671.88	0.92	1.08	1.01	f+p	f
0.9	0.25	0.54	0.3	0.15	7.09	0.20	18.9	413	255	813	818	235	801.79	874.29	870.56	1.09	1.01	0.94	f+s	f
0.9	0.25	0.54	0.3	0.15	7.09	0.20	22	413	235	813	813	260	801.79	874.29	963.32	0.90	1.01	0.92	f+p	f
0.8	0.2	0.5	0.3	0.15	4.25	0.15	29.8	405	225	490	500	228	379.58	413.90	879.43	0.99	1.28	1.20	f	f
0.8	0.2	0.5	0.3	0.15	4.25	0.15	29.8	405	235	490	495	228	379.58	413.90	879.43	1.03	1.28	1.19	f	f
0.8	0.3	0.5	0.3	0.15	5.67	0.25	28.9	405	392	1029	1039	375	851.77	928.79	1449.28	1.04	1.20	1.11	f+s	f
0.8	0.3	0.5	0.3	0.15	5.67	0.25	30.9	405	431	1029	1029	392	851.77	928.79	1515.40	1.10	1.20	1.10	f+s	f
0.8	0.3	0.5	0.25	0.15	5.67	0.25	29.1	405	363	833	853	377	851.77	928.79	1213.30	0.96	0.97	0.91	f+s	f
0.90	0.20	0.54	0.3	0.15	5.67	0.15	21.3	413	176	510	519	190	478.20	521.44	703.04	0.93	1.06	0.99	f+s	f
0.90	0.20	0.54	0.3	0.15	5.67	0.15	20.4	413	176	470	480	184	478.20	521.44	683.09	0.96	0.98	0.91	f+s	f
0.90	0.25	0.54	0.3	0.15	7.09	0.20	22.6	413	274	—	735	265	801.59	874.07	980.76	1.04	—	0.83	s	f
0.90	0.25	0.54	0.3	0.15	7.09	0.20	21.5	413	274	—	755	256	801.59	874.07	948.67	1.07	—	0.86	s	f
0.80	0.20	0.5	0.3	0.15	4.25	0.15	29.1	405	196	470	485	224	379.84	414.19	865.61	0.87	1.23	1.16	f+s	f
0.80	0.20	0.5	0.3	0.15	4.25	0.15	29.8	405	235	480	480	228	379.84	414.19	879.43	1.03	1.25	1.15	f+s	f
0.80	0.30	0.5	0.3	0.15	5.67	0.25	27.3	405	431	907	916	361	851.86	928.89	1395.29	1.19	1.06	0.98	s	f
0.80	0.30	0.5	0.3	0.15	5.67	0.25	28.5	405	392	1029	907	372	851.86	928.89	1435.88	1.05	1.20	0.97	f+s	f
0.80	0.30	0.5	0.25	0.15	5.67	0.25	30.9	405	402	784	794	392	851.86	928.89	1262.83	1.02	0.91	0.85	f+s	f
0.80	0.30	0.5	0.25	0.15	5.67	0.25	26.3	405	353	—	725	352	851.86	928.89	1134.17	1.00	—	0.77	s	f

Note: EFL = experimental failure load; AFL = analytical failure load; s = shear failure; f = flexural failure; f+s = flexure and shear failure; f+p = flexure and punching shear failure; 1 m = 39.37 in.; 1 cm^2 = 0.155 in.^{2}; 1 kN = 0.225 kip; and 1 MPa = 6.895 ksi.

Table 4—Comparison between proposed analytical model and experimental data from Suzuki et al.[17]

Geometrical properties, material properties, and experimental results												Predictions of proposed model				Comparisons				
L, m	h, m	e, m	$a = b$, m	p, m	A_{sD}, cm^2	d, m	f_c, MPa	f_y, MPa	$\|N_{c,e}\|$, kN	$\|N_{y,e}\|$, kN	$\|N_{f,e}\|$, kN	$\|N_{c,a}\|$, kN	$\|N_{y,a}\|$, kN	$\|N_{ff,a}\|$, kN	$\|N_{fs,a}\|$, kN	$N_{c,e}/N_{c,a}$	$N_{y,e}/N_{y,a}$	$N_{f,e}/N_{f,a}$	EFL	AFL
0.9	0.35	0.6	0.25	0.15	2.852	0.30	30.9	356	363	372	392	440	375.71	409.69	1511.82	0.82	0.98	0.95	f	f
0.9	0.35	0.6	0.25	0.15	2.852	0.30	28.2	356	372	372	392	414	375.71	409.69	1422.41	0.90	0.98	0.95	f	f
0.9	0.35	0.6	0.25	0.15	4.28	0.30	28.6	356	333	490	519	418	563.57	614.53	1435.83	0.80	0.86	0.84	f	f
0.9	0.35	0.6	0.25	0.15	4.28	0.30	28.8	356	314	470	472	420	563.57	614.53	1442.52	0.75	0.83	0.76	f	f
0.9	0.35	0.6	0.25	0.15	5.70	0.30	29.6	356	294	598	608	428	751.43	819.37	1469.11	0.69	0.79	0.74	f	f
0.9	0.35	0.6	0.25	0.15	5.70	0.30	29.3	356	255	578	627	425	751.43	819.37	1459.17	0.60	0.76	0.76	f	f
0.9	0.35	0.45	0.25	0.15	4.28	0.30	25.6	356	598	735	921	518	751.43	819.37	1333.58	1.15	0.97	1.12	f	f
0.9	0.35	0.45	0.25	0.15	4.28	0.30	27	356	559	725	833	537	751.43	819.37	1381.77	1.04	0.96	1.01	f	f
0.9	0.35	0.45	0.25	0.15	5.70	0.30	27.2	356	578	882	1005	539	1001.90	1092.50	1388.59	1.07	0.87	0.91	f	f
0.9	0.35	0.45	0.25	0.15	5.70	0.30	27.3	356	578	902	1054	541	1001.90	1092.50	1391.99	1.07	0.89	0.96	f	f
0.9	0.35	0.45	0.25	0.15	7.84	0.30	28	356	510	1117	1299	550	1377.61	1502.19	1415.68	0.93	0.80	0.92	f+s	s
0.9	0.35	0.45	0.25	0.15	7.84	0.30	28.1	356	529	1196	1303	551	1377.61	1502.19	1419.05	0.96	0.86	0.92	f+s	s
0.9	0.3	0.5	0.25	0.15	2.85	0.25	27.5	356	382	392	490	406	374.50	408.37	1161.90	0.94	1.04	1.19	f	f
0.9	0.3	0.5	0.25	0.15	2.85	0.25	26.3	356	363	392	461	394	374.50	408.37	1127.85	0.92	1.04	1.12	f	f
0.9	0.3	0.5	0.25	0.15	4.28	0.25	29.6	383	353	539	657	427	604.36	659.01	1220.32	0.83	0.88	0.99	f	f
0.9	0.3	0.5	0.25	0.15	4.28	0.25	27.6	383	372	549	657	407	604.36	659.01	1164.71	0.91	0.90	0.99	f	f
0.9	0.3	0.5	0.25	0.15	12.70	0.25	27	370	314	—	1245	399	1722.65	1878.43	1140.75	0.79	—	1.09	s	s

Note: EFL = experimental failure load; AFL = analytical failure load; s = shear failure; f = flexural failure; f+s = flexure and shear failure; 1 m = 39.37 in.; 1 cm^2 = 0.155 in.^{2}; 1 kN = 0.225 kip; and 1 MPa = 6.895 ksi.

The coefficient ϕ_y takes into account the yielding of reinforcements, whereas the coefficient ϕ_f takes into account the possibility of rupture of the longitudinal reinforcement. In this way, the relation connecting these coefficients should be the same relation connecting the yield strength and the rupture strength of the longitudinal reinforcements. As can be seen, the coefficient ϕ_f is 9% higher than the coefficient ϕ_y, which is approximately the same percentage obtained when dividing the minimum rupture strength by the yielding strength of a mild steel.

The proposed strut-and-tie model for predicting the behavior of four-pile caps is acceptably accurate considering the type of brittle and complex behavior associated with the response of three-dimensional pile caps. The coefficients of variation of the predictions of cracking and yielding of 0.14 and 0.15 are quite good, as shown in Table 7. While the

Table 5—Comparison between proposed analytical model and experimental data from Suzuki et al.[19]

Geometrical properties, material properties, and experimental results												Predictions of proposed model				Comparisons				
L, m	h, m	e, m	$a = b$, m	p, m	A_{sD}, cm^2	d, m	f_c, MPa	f_y, MPa	$\|N_{c,e}\|$, kN	$\|N_{y,e}\|$, kN	$\|N_{f,e}\|$, kN	$\|N_{c,a}\|$, kN	$\|N_{y,a}\|$, kN	$\|N_{ff,a}\|$, kN	$\|N_{fs,a}\|$, kN	$N_{c,e}/N_{c,a}$	$N_{y,e}/N_{y,a}$	$N_{f,e}/N_{f,a}$	EFL	AFL
0.70	0.20	0.45	0.25	0.15	2.84	0.15	26.1	358	186	284	294	201	246.80	269.12	665.72	0.92	1.14	1.08	f	f
0.70	0.20	0.45	0.25	0.15	2.84	0.15	26.1	358	206	294	304	201	246.80	269.12	665.72	1.02	1.18	1.12	f	f
0.80	0.20	0.45	0.25	0.15	2.84	0.15	25.4	358	225	304	304	226	246.80	269.12	653.77	1.00	1.22	1.12	f	f
0.80	0.20	0.45	0.25	0.15	2.84	0.15	25.4	358	225	284	304	226	246.80	269.12	653.77	1.00	1.14	1.12	f	f
0.90	0.20	0.45	0.25	0.15	2.84	0.15	25.8	358	235	323	333	257	246.80	269.12	660.61	0.92	1.30	1.23	f	f
0.90	0.20	0.45	0.25	0.15	2.84	0.15	25.8	358	235	333	333	257	246.80	269.12	660.61	0.92	1.34	1.23	f	f
0.70	0.30	0.45	0.2	0.15	4.25	0.25	25.2	358	333	510	534	332	624.64	681.12	877.83	1.00	0.81	0.78	f	f
0.70	0.30	0.45	0.2	0.15	4.25	0.25	24.6	358	353	500	549	326	624.64	681.12	863.84	1.08	0.79	0.80	f+s	f
0.80	0.30	0.45	0.2	0.15	4.25	0.25	25.2	358	382	519	568	379	624.64	681.12	877.83	1.01	0.82	0.83	f	f
0.80	0.30	0.45	0.2	0.15	4.25	0.25	26.6	358	372	529	564	393	624.64	681.12	910.05	0.95	0.84	0.82	f	f
0.90	0.30	0.45	0.2	0.15	4.25	0.25	26	358	421	559	583	435	624.64	681.12	896.31	0.97	0.89	0.85	f	f
0.90	0.30	0.45	0.2	0.15	4.25	0.25	26.1	358	421	539	588	436	624.64	681.12	898.61	0.96	0.86	0.86	f	f
0.70	0.30	0.45	0.25	0.15	4.25	0.25	28.8	383	343	647	662	362	668.26	728.69	1199.45	0.95	0.96	0.90	f+s	f
0.70	0.30	0.45	0.25	0.15	4.25	0.25	26.5	383	333	627	676	343	668.26	728.69	1134.71	0.97	0.93	0.92	f+s	f
0.80	0.30	0.45	0.25	0.15	4.25	0.25	29.4	383	431	657	696	420	668.26	728.69	1216.05	1.03	0.98	0.95	f+s	f
0.80	0.30	0.45	0.25	0.15	4.25	0.25	27.8	383	421	686	725	405	668.26	728.69	1171.52	1.04	1.02	0.99	f+s	f
0.90	0.30	0.45	0.25	0.15	4.25	0.25	29	383	470	666	764	468	668.26	728.69	1205.00	1.00	0.99	1.04	f+s	f
0.90	0.30	0.45	0.25	0.15	4.25	0.25	26.8	383	461	657	764	444	668.26	728.69	1143.26	1.04	0.98	1.04	f	f
0.70	0.30	0.45	0.3	0.15	4.25	0.25	26.8	383	461	745	769	345	668.26	728.69	1371.91	1.33	1.11	1.05	f+s	f
0.70	0.30	0.45	0.3	0.15	4.25	0.25	25.9	358	451	676	730	338	624.64	681.12	1341.02	1.34	1.07	1.06	f+s	f
0.80	0.30	0.45	0.3	0.15	4.25	0.25	27.4	358	490	735	828	401	624.64	681.12	1392.31	1.22	1.17	1.21	f+s	f
0.80	0.30	0.45	0.3	0.15	4.25	0.25	27.4	358	480	745	809	401	624.64	681.12	1392.31	1.20	1.18	1.18	f+s	f
0.90	0.30	0.45	0.3	0.15	4.25	0.25	27.2	358	568	764	843	449	624.64	681.12	1385.53	1.27	1.21	1.23	f+s	f
0.90	0.30	0.45	0.3	0.15	4.25	0.25	24.5	358	490	745	813	418	624.64	681.12	1292.25	1.17	1.18	1.18	f+s	f
0.70	0.40	0.45	0.25	0.15	5.67	0.35	25.9	358	519	—	1019	475	1172.10	1278.08	1572.72	1.09	—	0.79	s	f
0.70	0.40	0.45	0.25	0.15	5.67	0.35	24.8	358	549	1039	1068	462	1172.10	1278.08	1527.87	1.19	0.88	0.83	f+s	f
0.80	0.40	0.45	0.25	0.15	5.67	0.35	26.5	358	598	1058	1117	551	1172.10	1278.08	1596.92	1.08	0.90	0.87	f	f
0.80	0.40	0.45	0.25	0.15	5.67	0.35	25.5	358	657	1088	1117	537	1172.10	1278.08	1556.48	1.22	0.92	0.87	f+s	f
0.90	0.40	0.45	0.25	0.15	5.67	0.35	25.7	358	715	1068	1176	608	1172.10	1278.08	1564.61	1.18	0.90	0.91	f	f
0.90	0.40	0.45	0.25	0.15	5.67	0.35	26	358	706	1117	1181	613	1172.10	1278.08	1576.76	1.15	0.95	0.92	f	f

Note: EFL = experimental failure load; AFL = analytical failure load; s = shear failure; f = flexural failure; f+s = flexure and shear failure; 1 m = 39.37 in.; 1 cm^2 = 0.155 in.^{2}; 1 kN = 0.225 kip; and 1 MPa = 6.895 ksi.

Table 6—Comparison between proposed analytical model and experimental data from Suzuki and Otsuki[16]

Geometrical properties, material properties, and experimental results											Predictions of proposed model				Comparisons			
L, m	h, m	e, m	$a = b$, m	p, m	A_{sD}, cm^2	d, m	f_c, MPa	f_y, MPa	$\|N_{y,e}\|$, kN	$\|N_{f,e}\|$, kN	$\|N_{c,a}\|$, kN	$\|N_{y,a}\|$, kN	$\|N_{ff,a}\|$, kN	$\|N_{fs,a}\|$, kN	$N_{y,e}/N_{y,a}$	$N_{f,e}/N_{f,a}$	EFL	AFL
0.8	0.35	0.5	0.3	0.15	6.42	0.29	24.1	353	—	960	391	987.99	1077.33	1509.83	-	0.88	s	f
0.8	0.35	0.5	0.3	0.15	6.42	0.29	25.6	353	—	941	407	987.99	1077.33	1571.84	-	0.87	s	f
0.8	0.35	0.5	0.3	0.15	6.42	0.29	23.7	353	1019	1029	387	987.99	1077.33	1493.07	1.02	0.95	f+s	f
0.8	0.35	0.5	0.3	0.15	6.42	0.29	23.5	353	1098	1103	384	987.99	1077.33	1484.66	1.10	1.02	f+s	f
0.8	0.35	0.5	0.3	0.15	6.42	0.29	31.5	353	—	980	467	987.99	1077.33	1804.91	—	0.90	s	f
0.8	0.35	0.5	0.3	0.15	6.42	0.29	32.7	353	1078	1088	479	987.99	1077.33	1850.47	1.08	1.00	f+s	f
0.8	0.35	0.5	0.25	0.15	6.42	0.29	27.1	353	892	902	423	987.99	1077.33	1360.55	0.90	0.83	f+s	f
0.8	0.35	0.5	0.25	0.15	6.42	0.29	25.6	353	—	872	407	987.99	1077.33	1309.87	—	0.80	s	f
0.8	0.35	0.5	0.25	0.15	6.42	0.29	23.2	353	902	911	381	987.99	1077.33	1226.67	0.91	0.84	f+s	f
0.8	0.35	0.5	0.25	0.15	6.42	0.29	23.7	353	882	921	387	987.99	1077.33	1244.23	0.89	0.85	f+s	f
0.8	0.35	0.5	0.25	0.15	6.42	0.29	36.6	353	—	882	517	987.99	1077.33	1662.35	—	0.81	s	f
0.8	0.35	0.5	0.25	0.15	6.42	0.29	37.9	353	—	951	529	987.99	1077.33	1701.48	—	0.88	s	f
0.8	0.35	0.5	0.2	0.15	6.42	0.29	22.5	353	—	755	374	987.99	1077.33	961.49	—	0.79	s	s
0.8	0.35	0.5	0.2	0.15	6.42	0.29	21.5	353	—	735	362	987.99	1077.33	932.79	—	0.79	s	s
0.8	0.35	0.5	0.2	0.15	6.42	0.29	20.4	353	745	755	350	987.99	1077.33	900.70	0.75	0.84	f+p	s
0.8	0.35	0.5	0.2	0.15	6.42	0.29	20.2	353	784	804	348	987.99	1077.33	894.80	0.79	0.90	f+s	s
0.8	0.35	0.5	0.2	0.15	6.42	0.29	31.4	353	—	813	466	987.99	1077.33	1200.73	—	0.75	s	f
0.8	0.35	0.5	0.2	0.15	6.42	0.29	30.8	353	—	794	460	987.99	1077.33	1185.38	—	0.73	s	f

Note: EFL = experimental failure load; AFL = analytical failure load; s = shear failure; f = flexural failure; f+s = flexure and shear failure; f+p = flexure and punching shear failure; 1 m = 39.37 in.; 1 cm^2 = 0.155 in.^{2}; 1 kN = 0.225 kip; and 1 MPa = 6.895 ksi.

Table 7—Precision of proposed analytical model based on experimental results[14-19]

	Experimental loads/analytical predictions		
	Cracking	Yielding	Failure
Average	1.00	1.01	1.01
Coefficient of variation	0.14	0.15	0.23
Variance	0.02	0.02	0.05
Standard deviation	0.14	0.15	0.23

coefficient of variation for failure load is 0.23, this is still considered low, as it is below the coefficient of variation for the ACI expression for the shear strength of slender beams that do not contain shear reinforcement.[26] The results presented in Tables 1 to 6 also illustrate that the model is able to successfully predict the failure mode with 87% of the failure modes of 129 four-pile caps being correctly predicted.

CONCLUDING REMARKS

Codes of practice do not provide clear guidance for the design of pile caps, and for this reason, many designers apply empirical approximations or rules of thumb for the design of this important component of structures. Most codes of practice recommend that pile caps be designed using sectional force methods and the application of sectional design methods. This is not appropriate for the majority of pile caps that are very stocky and for which direct compression struts flow from the point of load application to the pile, thereby creating a complex and nonlinear strain distribution throughout the pile cap.

The results of previous research[9-13] have illustrated that traditional shear provisions can be quite unconservative when applied to pile caps; load tests on pile caps designed to fail in flexure resulted in shear failures.[14-19] The main reason for these early shear failures can be explained by an exaggerated importance given to the effective depth when applying sectional design methods.

The strut-and-tie method is an alternative and appropriate design procedure for pile caps that is now supported in some codes of practice.[1,4,27] Although additional shear verification is not recommended in codes when using the strut-and-tie method, it is strongly advised when designing pile caps. It is believed that shear failure in pile caps is the result of compression struts splitting longitudinally. To prevent this sort of failure, a compressive stress under $1.0f_c$ and a relation shear span-depth ratio under 1.0 normally can lead to ductile failures. In this way, one may expect the yielding of longitudinal reinforcement prior to the crushing or splitting of compression struts.

While some research illustrates that the use of a strut-and-tie method will lead to the use of less longitudinal reinforcement to support the same loading by approximately 10 to 20%,[12,15] Nori and Tharval[28] concluded exactly the opposite situation. This inconsistency may be explained focusing on two major factors: the position of the critical section for bending (sectional approach) and the position of the nodal zone underneath the column (strut-and-tie method). For that reason, one can observe that is difficult to generalize the fact that strut-and-tie is more economical than the sectional approach, although it can provide a more rational and safe method for proportioning the depth of pile caps for shear.

Souza et al.[24] presented an adaptable three-dimensional strut-and-tie model that can be applied to the design or analysis of four-pile caps supporting rectangular columns that impose compressive loads and biaxial flexure onto the top of the pile cap. This is a very common situation that is usually not treated in most codes, and for that reason, designers have been applying the bending theory or a simplified strut-and-tie model[14] developed for the condition of square columns subjected to axial load.

Simplifying the aforementioned model[24] for the simple case of axial loads and square columns, and using an experimental test database, equations were developed and calibrated for predicting the cracking, yielding, and failure load of four-pile caps as well as their mode of failure. The calibrated model has coefficients of variation of 14, 15, and 23% for predicting cracking, yielding, and failure loads and was successful in predicting the mode of failure in 87% of cases. Because these methods were developed to fit a large and broad array of pile cap dimensions, reinforcement conditions, and span-depth ratios, they are expected to be generally applicable for most common design situations.

An evaluation of the experimental test data further revealed that the tensile contribution of the concrete is underestimated by the application of either the sectional design methods or strut-and-tie design provisions for very stocky pile caps (shear span-depth ratios < 0.60). Thus, the expected failure load in these stocky members is expected to be considerably larger than the factored design load. In many cases, stocky pile caps were found to be able to carry their factored design loads prior to cracking and thus the method used for the design of longitudinal reinforcement is not evaluated by field experience.

The paper also illustrated that the use of the sectional design methods can be an inadequate procedure for stocky pile caps. Coupled with observations that the shear provisions can be unconservative, pile caps designed by this sectional philosophy are likely to exhibit brittle failures if overloaded. Strut-and-tie models better represent the flow of forces in pile caps but improved models are needed that can account for compatibility and the nonlinear behavior and tensile contributions of concrete materials.

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NOTATION

A_{sT}, A_{sD} = total amount of bunched and grid reinforcement regarding one direction
a, b = column dimensions
$C_{A,k}$, $C_{B,k}$, $C_{C,k}$, $C_{D,k}$ = nominal forces acting in struts, A, B, C, and D, respectively
c = average distance between face column and pile centers
c/d = span-depth ratio
cx, cy = distance between face columns and pile center in *x*- and *y*-directions, respectively
d = effective depth of pile caps
EFM, AFM = failure mode reported experimentally and predicted using proposed model, respectively
e = pitch between center of piles
$e_{k,x}$, $e_{k,y}$ = nominal eccentricities of load from *x*- and *y*-axes
f_c, f_y = concrete compression strength and steel yielding strength, respectively
f_t = concrete tensile strength
L = pile cap length and width
M_{kx}, M_{ky} = nominal flexure loading acting from column on pile cap about *x*- and *y*-axes

$N_{c,a}$, $N_{y,a}$, $N_{f,a}$ = analytical cracking, yielding, and failure loads, respectively
$N_{c,e}$, $N_{y,e}$, $N_{f,e}$ = experimental cracking, yielding, and failure loads, respectively
$N_{ff,a}$, $N_{fs,a}$ = analytical failure load for flexure and shear, respectively
N_k = nominal axial loading acting on column
p = pile diameter or width
$R_{A,k}$, $R_{B,k}$, $R_{C,k}$, $R_{D,k}$ = nominal pile reaction force for piles, A, B, C, and D, respectively
$T_{AC,k}$, $T_{BD,k}$, $T_{CD,k}$, $T_{AB,k}$ = nominal forces acting in ties, AC, BD, CD, and AB, respectively
tg, tg^{-1}, sin, cos = tangent, cotangent, sine, and cosine, respectively
ϕ_c, ϕ_y, ϕ_f = coefficient of calibration for cracking, yielding, and failure, respectively
γ, ϕ = safety factor for loads and strength reduction factor for materials, respectively
θ_A, θ_B, θ_C, θ_D = angles between struts and ties, A, B, C and D, respectively
$\sigma_{f,e}$, $\tau_{f,e}$ = normalized normal stress for the columns and normalized shear strength at experimental failure
ω = mechanical reinforcement ratio

REFERENCES

1. ACI Committee 318, "Building Code Requirements for Structural Concrete (ACI 318-05) and Commentary (318R-05)," American Concrete Institute, Farmington Hills, MI, 2005, 430 pp.

2. British Standards Institution, "Code of Practice for the Design of Concrete Bridges—BS 5400," Milton Keynes, UK, 1990.

3. British Standards Institution, "Structural Use of Concrete: BS 8110," Milton Keynes, UK, 1997.

4. Canadian Standards Association, "CSA Standard-A23.3-94—Design of Concrete Structures," Rexdale, ON, Canada, 1994.

5. ACI Committee 318, "Building Code Requirements for Reinforced Concrete (ACI 318-83)," American Concrete Institute, Farmington Hills, MI, 1983.

6. ACI Committee 318, "Building Code Requirements for Structural Concrete (ACI 318-99) and Commentary (318R-99)" American Concrete Institute, Farmington Hills, MI, 1999, 391 pp.

7. ACI Committee 318, "Building Code Requirements for Structural Concrete (ACI 318-02) and Commentary (318R-02)," American Concrete Institute, Farmington Hills, MI, 2002, 443 pp.

8. Schlaich, J.; Schafer, K.; and Jennewein, M., "Toward a Consistent Design of Reinforced Concrete Structures," *Journal of Prestressed Concrete Structures*, V. 32, No. 3, 1987, pp. 74-150.

9. Adebar, P.; Kuchma, D.; and Collins, M. P., "Strut-and-Tie Models for the Design of Pile Caps: An Experimental Study," *ACI Structural Journal*, V. 87, No. 1, Jan.-Feb. 1990, pp. 81-92.

10. Adebar, P., and Zhou, Z., "Design of Deep Pile Caps by Strut-and-Tie Models," *ACI Structural Journal*, V. 93, No. 4, July-Aug. 1996, pp. 1-12.

11. Bloodworth, A. G.; Jackson, P. A.; and Lee, M. M. K., "Strength of Reinforced Concrete Pile Caps," *Proceedings of the Institution of Civil Engineers*, Structures & Buildings 156, Issue SB4, Nov. 2003, pp. 347-358.

12. Cavers, W., and Fenton, G. A., "An Evaluation of Pile Cap Design Methods in Accordance with the Canadian Design Standard," *Canadian Journal of Civil Engineering*, V. 31, No. 1, Feb. 2004, pp. 109-119.

13. Park, J.; Kuchma, D. A.; and Souza, R. A., "Strength Predictions of Pile Caps by a Strut-and-Tie Model Approach," *Canadian Journal of Civil Engineering*, V. 35, No. 12, 2008, pp. 1399-1413.

14. Blévot, J. L., and Frémy, R., "Semelles sur Pieux," *Institute Technique du Bâtiment et des Travaux Publics*, V. 20, No. 230, 1967, pp. 223-295.

15. Clarke, J. L., "Behavior and Design of Pile Caps with Four Piles," *Technical Report* No. 42.489, Cement and Concrete Association, Wexham Springs, 1973.

16. Suzuki, K., and Otsuki, K., "Experimental Study on Corner Shear Failure of Pile Caps," *Transactions of the Japan Concrete Institute*, V. 23, 2002.

17. Suzuki, K.; Otsuki, K.; and Tsubata, T., "Experimental Study on Four-Pile Caps with Taper," *Transactions of the Japan Concrete Institute*, V. 21, 1999, pp. 327-334.

18. Suzuki, K.; Otsuki, K.; and Tsubata, T., "Influence of Bar Arrangement on Ultimate Strength of Four-Pile Caps," *Transactions of the Japan Concrete Institute*, V. 20, 1998, pp. 195-202.

19. Suzuki, K.; Otsuki, K.; and Tsuhiya, T., "Influence of Edge Distance on Failure Mechanism of Pile Caps," *Transactions of the Japan Concrete Institute*, V. 22, 2000, pp. 361-367.

20. Comité Euro-International Du Béton, "CEB-FIP International Recommendations for the Design and Construction of Concrete Structures," London, UK, 1970.

21. British Standards Institution, "Code of Practice for the Structural Use of Concrete—CP 110," London, UK, 1972.

22. Sabnis, G. M., and Gogate, A. B., "Investigation of Thick Slabs (Pile Caps)," ACI JOURNAL, *Proceedings* V. 81, No. 1, Jan.-Feb. 1984, pp. 35-39.

23. Adebar, P., and Zhou, Z., "Bearing Strength of Compressive Struts Confined by Plain Concrete," *ACI Structural Journal*, V. 90, No. 5, Sept.-Oct. 1993, pp. 534-541.

24. Souza, R. A.; Kuchma, D. A.; Park, J.; and Bittencourt, T. N., "Non-Linear Finite Element Analysis of Four-Pile Caps Supporting Columns Subjected to Generic Loading," *Computers and Concrete*, V. 4, No. 5, 2007, pp. 363-376.

25. Siao, W. B., "Strut-and-Tie Model for Shear Behavior in Deep Beams and Pile Caps Failing in Diagonal Splitting," *ACI Structural Journal*, V. 90, No. 4, July-Aug. 1993, pp. 356-363.

26. Reineck, K. H.; Kuchma, D. A.; Kim, K. S.; and Marx, S., "Shear Database for Reinforced Concrete Members without Shear Reinforcement," *ACI Structural Journal*, V. 100, No. 2, Mar.-Apr. 2003, pp. 240-249.

27. Comité Euro-International Du Béton, "CEB-FIP Model Code 1990," Thomas Telford Services, Ltd., London, UK, 1993.

28. Nori, V. V., and Tharval, M. S., "Design of Pile Caps—Strut and Tie Method," *Indian Concrete Journal*, V. 81, No. 4, 2007, pp. 13-19.

29. Canadian Portland Cement Association, *Concrete Design Handbook*, Ottawa, ON, Canada, 1995.

30. Cervenka, V.; Jendele, L.; and Cervenka, J., "ATENA Program Documentation—Part 1: Theory," Praha, Czech Republic, 2005.